Baryonium in Confining Gauge Theories

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Baryon in gravity dual


Baryon in SU(N) gauge theory

... the strings stretched between the boundary of the $AdS_5$ and the D5-brane wrapped on the $S^5$

E. Witten (‘98)
In the type IIB string theory, there is a self dual field strength $G_5$ and there are $N$ units of flux on the $S^5$

$$\int_{S^5} \frac{G_5}{2\pi} = N$$

On the D5-brane, there is a $U(1)$ gauge field $A$ which couples to $G_5$ as

$$\int_{R \times S^5} A \wedge \frac{G_5}{2\pi}$$

By adding $(-1)$ unit of charge to each string endpoint, the total charge in $S^5$ vanishes.
**deformed AdS**

We consider the deformed $AdS$ background by including the R-R scalar $\chi$

$$S = \frac{1}{2\kappa^2} \int d^{10} x \sqrt{g} \left( R - \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} e^{2\phi} (\partial \chi)^2 - \frac{1}{4 \cdot 5!} F_5^2 \right)$$

the ansatz for supersymmetry: $\chi = -e^{\phi} + \chi_0$  
(A. Kehagius and K. Sfetsos ('99))

The solution is given as follows.

$$d s^2 = e^{\phi} \left( \frac{r^2}{R^2} (-dt^2 + (dx^i)^2) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right)$$

$$e^{\phi} = 1 + \frac{q}{r^4} \quad q \equiv \frac{\chi_0 R}{4} \quad R = (4\pi g_s \alpha'^2 N)^{1/4}$$

$q \cdots$ VEV of gauge field condensate $\langle F^{\mu\nu} F_{\mu\nu} \rangle$ (H. Liu and A.A. Tseytlin('99))
The dual gauge theory of this background is in the confinement phase.

The $q\bar{q} - $potential is given by the energy of the U-shaped string.

The U-shaped configuration

$X_{||} = (\sigma, 0, 0), \quad \Omega_5 = \text{constant}$
D5-brane action in this deformed $AdS_5 \times S^5$ background is

$$S = -T_5 \int d^6 \xi \ e^{-\Phi} \sqrt{-\det(g + F)} + T_5 \int A_{(1)} \wedge G_{(5)},$$

where, $G_5$ is self-dual Ramond-Ramond field strength

$$G_{(5)} \equiv dC_{(4)} = 4R^4 \left( \text{vol}(S^5) d\theta_1 \wedge \ldots \wedge d\theta_5 - \frac{r^3}{R^8} dt \wedge \ldots \wedge dx_3 \wedge dr \right)$$

where $\text{vol}(S^5) \equiv \sin^4 \theta_1 \text{vol}(S^4) \equiv \sin^4 \theta_1 \sin^3 \theta_2 \sin^2 \theta_3 \sin \theta_4$

and $F_{(2)} = dA_{(1)}$ is the U(1) worldvolume field strength on the D5-brane.
We set the world volume coordinates of D5-brane as 
\[ \xi^a = (t, \theta, \theta_2, \ldots, \theta_5). \]
\( r, \) and \( A_t \) depend only on \( \theta \). (C.G. Callan, et al. ('99) 
Y. Imamura ('99) M.I and K. Ghoroku ('08))

Then, the embedded configuration of the D5 brane is

\[
S = T_5 \Omega_4 R^4 \int dtd\theta \sin^4 \theta (-\sqrt{e^\Phi (r^2 + r'^2) - F_{\theta t}^2 + 4A_t})
\]

where, \( \Omega_4 = \frac{8\pi^2}{3} \) is the volume of the unit four sphere.

We define the dimensionless displacement as

\[
D = \frac{1}{T_5 \Omega_4 R^4} \frac{\delta S}{\delta F_{t\theta}}
\]

Then, the gauge field equation of motion and its solution are given.

\[
\partial_\theta D = -4 \sin^4 \theta
\]

\[
D \equiv D(\nu, \theta) = \left[ \frac{3}{2}(\nu\pi - \theta) + \frac{3}{2} \sin \theta \cos \theta + \sin^3 \theta \cos \theta \right]
\]
We eliminate the gauge filed in favor of $D$.

\[
U_{D5} = \frac{N}{3 \pi^2 \alpha'} \int d\theta \ e^{\Phi/2} f \sqrt{r^2 + r'^2} \sqrt{V_{\nu}(\theta)}
\]

\[
V_{\nu}(\theta) = D(\nu, \theta)^2 + \sin^8 \theta
\]

where we used $T_5 \Omega_4 R^4 = N/(3 \pi^2 \alpha')$.

Then, we get the equation of motion

\[
\partial_\theta \left( \frac{r'}{\sqrt{r^2 + (r')^2}} \sqrt{V_{\nu}(\theta)} \right) - \frac{1 - q/r^4}{1 + q/r^4} \frac{r}{\sqrt{r^2 + (r')^2}} \sqrt{V_{\nu}(\theta)} = 0
\]
The left hand side shows the solutions for $q = 2.8$ and $\nu = 0$
The right hand side shows the solutions for $\nu = 0.2$

These solutions have cusps.
two cusps for $\nu \neq 0$ at $\theta = \pi$ and $\theta = 0$
one cusp for $\nu = 0$ at $\theta = \pi$
For simplicity, we consider the case of $\nu = 0$ in this section.
The D5-brane configuration is singular at \( r(\theta = \pi) \) for \( \nu = 0 \). The flux number at this cusp is given as

\[
\frac{\delta S}{\delta F_{t\theta}}|_{\theta=\pi} = \frac{2N}{3\pi} D(\pi) = N \quad \text{(here we set} 2\pi\alpha' \equiv 1 \text{)}
\]

Then, this singularity is canceled out by the boundary term of the \( N \) F-strings stretching from this cusp. We can get the baryon.
baryonium

We set its world volume coordinates as $\xi^a = (t, \theta, \theta_2, \ldots, \theta_5)$. $r, A_t$ and $x$ depend only on $\theta$. (C.G.Callan, et al. ('99) M.I and K. Ghoroku ('08))

Then, the embedded configuration of the D5 brane is

$$S = T_5 \Omega_4 R^4 \int dt d\theta \sin^4 \theta (-\sqrt{e^\Phi (r^2 + r'^2 + (r/R)x'^2)} - F_{\theta t}^2 + 4 A_t)$$

We eliminate the gauge filed in favor of $D$.

$$U_{D5} = \frac{N}{3\pi^2 \alpha'} \int d\theta \ e^{\Phi/2} f \sqrt{r^2 + r'^2 + (r/R)^4 x'^2} \sqrt{V_\nu(\theta)}$$

$$V_\nu(\theta) = D(\nu, \theta)^2 + \sin^8 \theta$$

where we used $T_5 \Omega_4 R^4 = N/(3\pi^2 \alpha')$. 
The typical $D5/\overline{D5}$ solution for $\nu = 0.5$. 
There are two cusps at $\theta = \pi$. The flux numbers at cusps are given as

$$\left| \frac{\delta S}{\delta F_{t\theta}} \right|_{\theta=\pi} = \frac{2N}{3\pi} |D(\nu, \pi)| = N - k \quad (k \equiv N \nu)$$

We cannot decide the sign of $D$ because $U_{D5}$ contains $D$ in the squared form $D^2$. However, in order to preserve the flux, $D$ should have opposite sign at the two cusps as

$$\pm \left. \frac{\delta S}{\delta F_{t\theta}} \right|_{\theta=\pi} = \frac{2N}{3\pi} D(\nu, \pi, x^\pm) = \pm (N - k)$$

where, $x^\pm = \pm x(\theta = \pi)$

Then, this solution can be interpreted as the $D5/\bar{D}5$ configuration.
\((N - k)\) F-strings must be attached at both cusps but with opposite orientation to cancel out the singularity at the two cusps. Such a state forms the \((N - k)\) quarks and \((N - k)\) anti-quarks bound state.
left: the $U_{D5} - L$ relation for $h=-1$ \ ($L \equiv x^+ - x^-$) \ 

The energy of vertex part of a baryonium ($U_{D5}$) has a minimum at finite $L = L_*$. When we can set the parameters to satisfy the inequality \ 

$$L_*^2/R^2 > \frac{1}{2\pi \alpha'}$$ \ 

The baryonium configuration is stable against the tachyon which would appear as strings connecting between D5 and anti-D5 branes.
Summary and future work

• We find a baryonium solution by solving the equation of motion for the D5-brane action.

• Since the equation of D5 action contains the displacement flux operator $D$ in the squared form $D^2$, the $D5/D\bar{D}5$ bound state is obtained from the D5-brane action.

• There is a minimum of $U_{D5}$ at finite $L = L_*$. Then, $D5$ and $D\bar{D}5$ are separated enough not to be destabilized by the tachyon. Then, the baryonium state found here would be stable.

• As for the total mass of the baryonium, we must add F-strings at the cusps of D5-brane. The detail of the analyses will given in the future.