虚数化学ポテンシャルを利用した現象論的模型による QCD相図の解析

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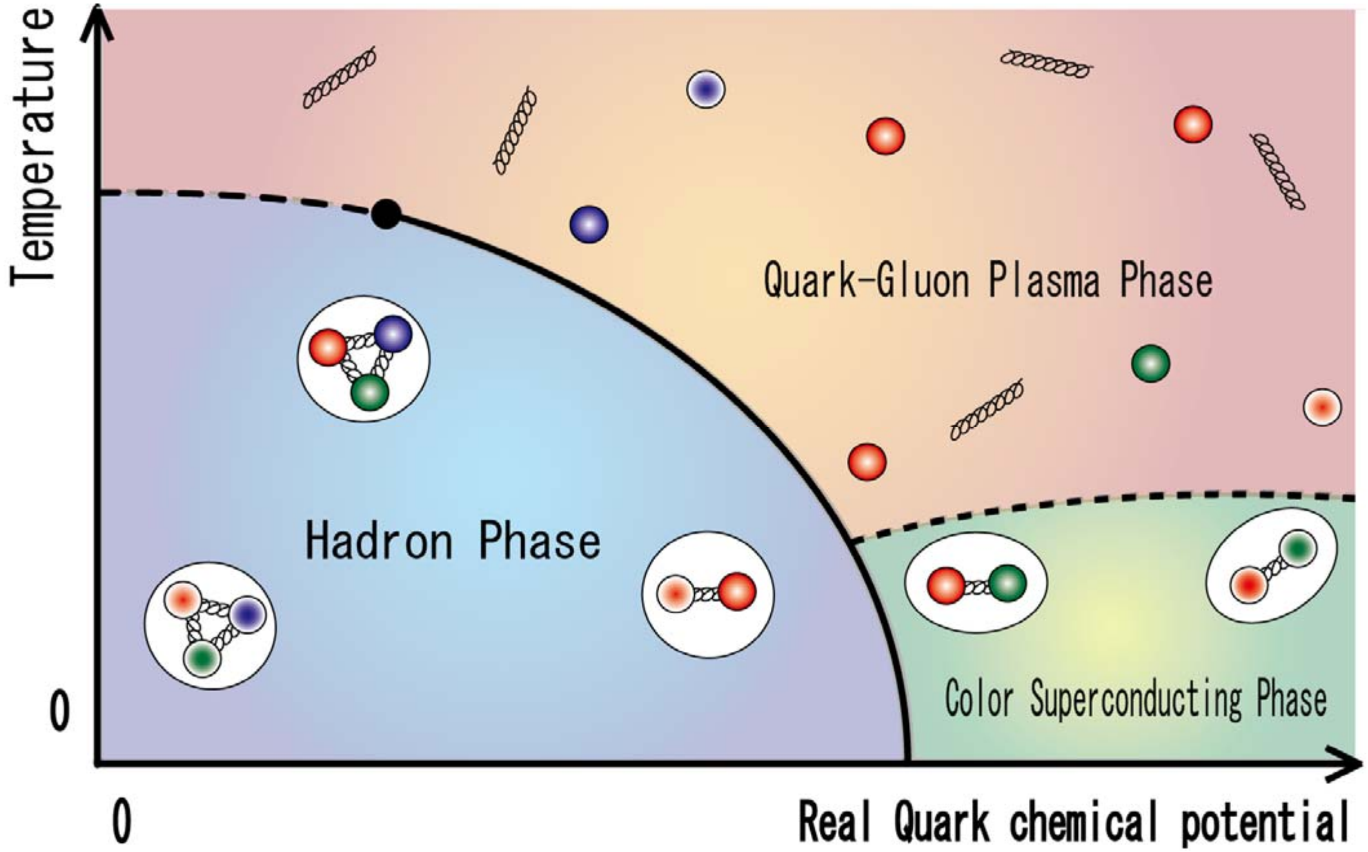
Objects of our study

Phase structure of
Quantum Chromodynamics
at **finite temperature** and **density**

A schematic view



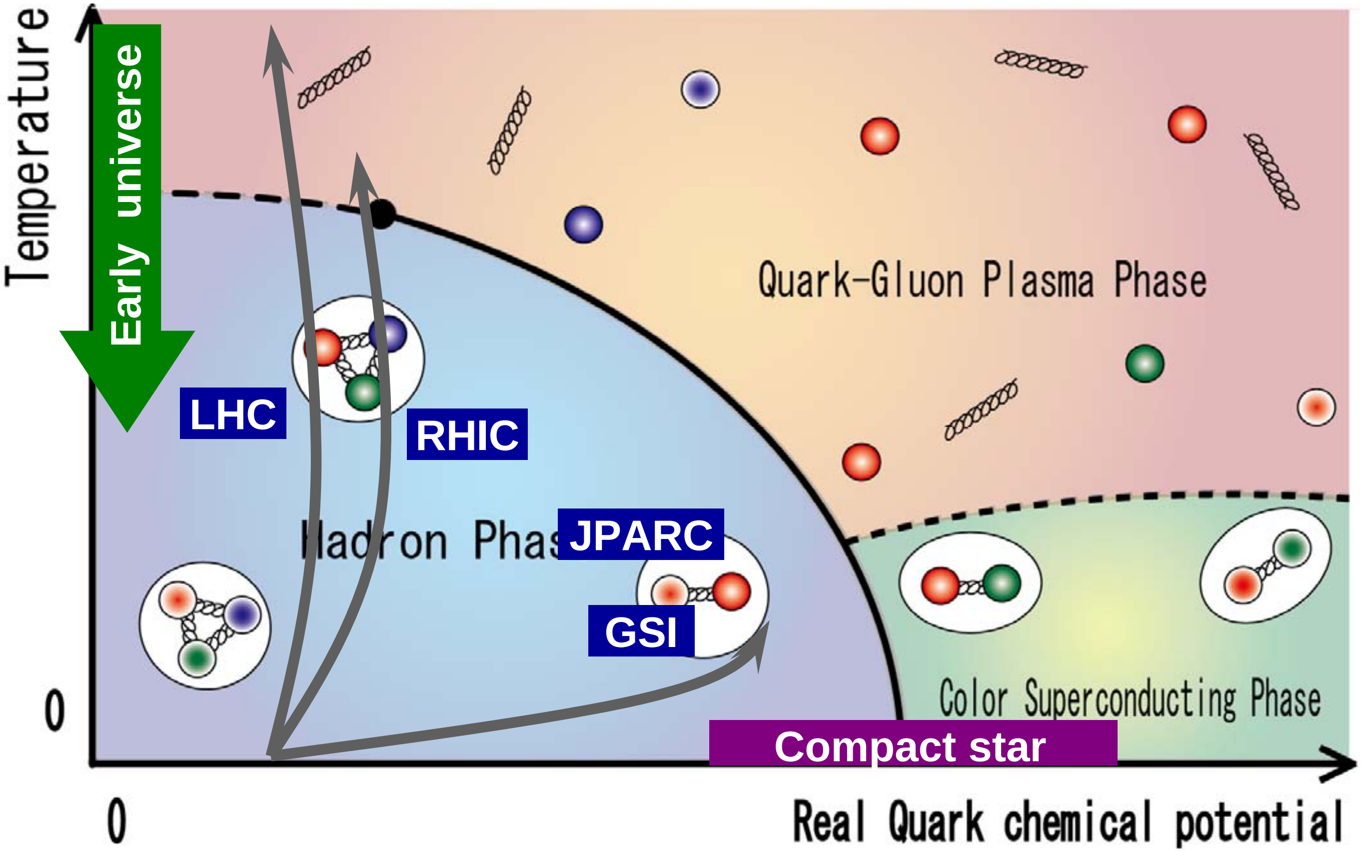
Importance

Study of QCD phase diagram is **important subject**

because it deeply related to **experiment** and the study of **heavy star**.

A schematic view

QCD phase structure is one of the recent hot topics.



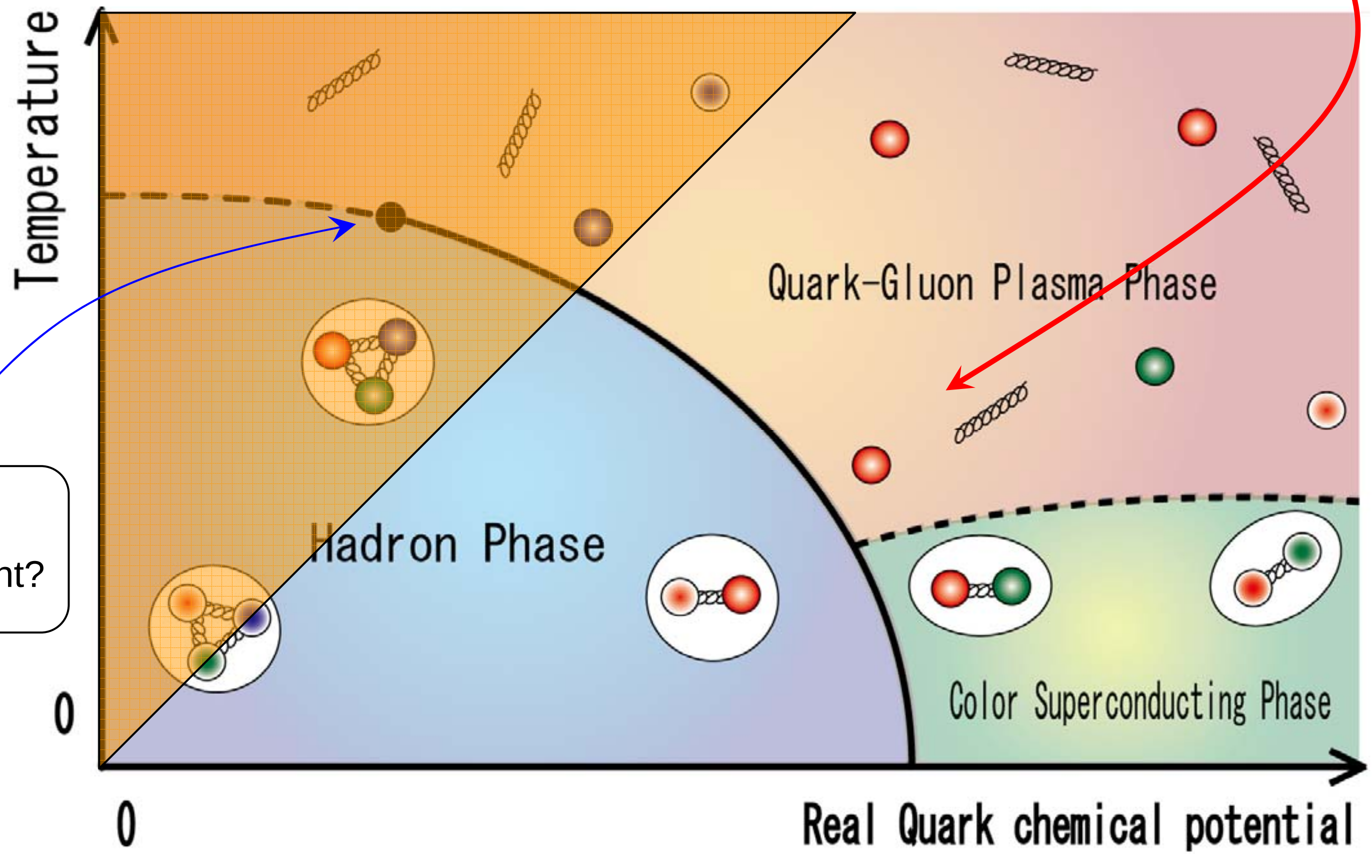
Problem 1

Sign problem

First principle calculation **can not** worked **at finite real chemical potential** (Lattice QCD)

LQCD can be calculated inside of below triangle if we use the approximation.

We want to know these finite density region exactly



Where is the critical end point?

Sign problem

Sign problem

Lattice QCD can calculate the QCD exactly
at zero chemical potential.

- But, calculation time is **very long**. $\left[\ (12-32)^3\times 4,\ 16^3\times 8\ \ldots\ \right]$

  ⟶ Development of computer power is necessary !!

- Moreover, lattice QCD has the **sign problem** at real chemical potential.

Important Method

Important sampling

$$Z(\mu_q)=\int \mathscr{D}\mathrm{U}\,\det[\mathscr{M}]\ \exp\left(-S_G\right)$$

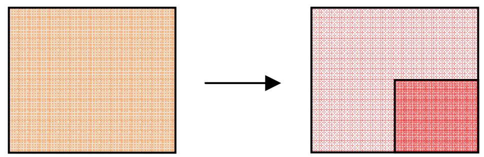
$$\left\langle \mathscr{O}\right\rangle=\frac{1}{Z}\int \mathscr{D}\mathrm{U}\ \mathscr{O}\,\underline{\det[\mathscr{M}]\ \exp\left(-S_G\right)}$$

probability

If the probability
become complex and
non-positive definite,
simulation does not work.

Dirac operator :

$$\mathscr{M}(\mu_q)=\gamma_\mu D^\mu+\gamma_4\mu_q+m_0$$



Statistical dynamics

Partition function

$$Z(\tau)=\sum_i \exp(-\beta\varepsilon_i)$$

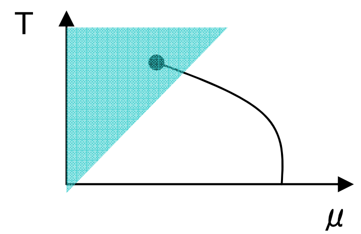
Probability in this case

$$P(\varepsilon_1)=\frac{\exp(-\beta\varepsilon_1)}{Z(\tau)}\le 1$$

LQCD Approach

Finite density

Therefore, some approaches are suggested.



- **Taylor expansion method**
  ex.) C. R. Allton et al., Phys. Rev. D **66**, 074507 (2002); Phys. Rev. D **68**, 014507 (2003).

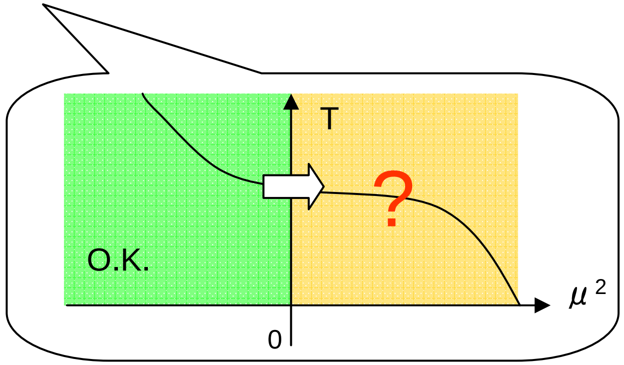
- **Reweighting method**
  ex.) Z. Fodor et al., Phys. Lett. B **534**, 87 (2002); J. High Energy Phys. **03**, 014 (2002); **04**, 050 (2004).

Other problem appeared.

For example, Taylor expansion method can be done at $\mu/T<1$ region only. (It is unreliable near the $T=\mu$)

- **Imaginary chemical potential.**
  ex.) P. De Forcrand et al., Nucl. Phys. **B642**, 290 (2002).
  M. D'Elia et al., Phys. Rev. D **67**, 014505 (2003).

It is not so reliable at real chemical potential. because the extrapolation is used.



- **Two color**
  ex.) C. Ratti, and W. Weise, Phys. Rev. D **70** (2004) 054013.
  K. Fukushima and K. Iida, Phys. Rev. D **76** (2007) 054004.

There are no sign problem.
But it is not realistic case.
(There are color singlet diquark exist.)

Those are far from perfection!!

Problem 2-1a

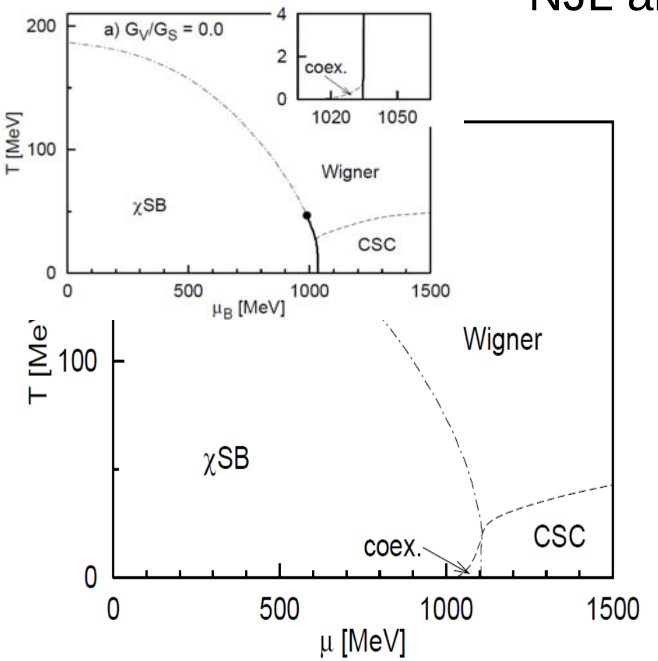
Ambiguity

Is there the critical point on QCD phase diagram?

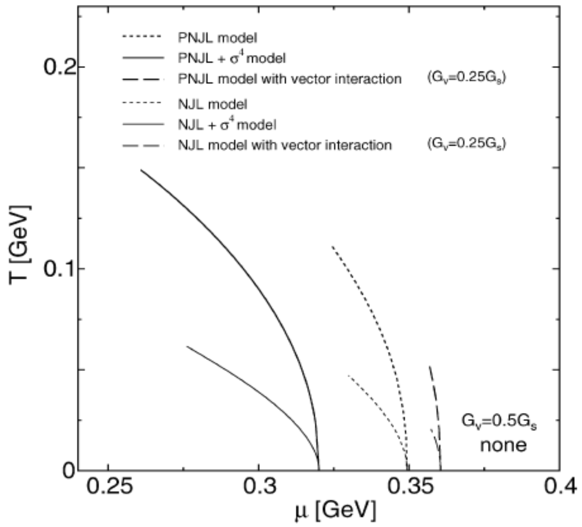
Effective models have **ambiguity** in the interaction part.

Vector type interaction is essential at **high density**

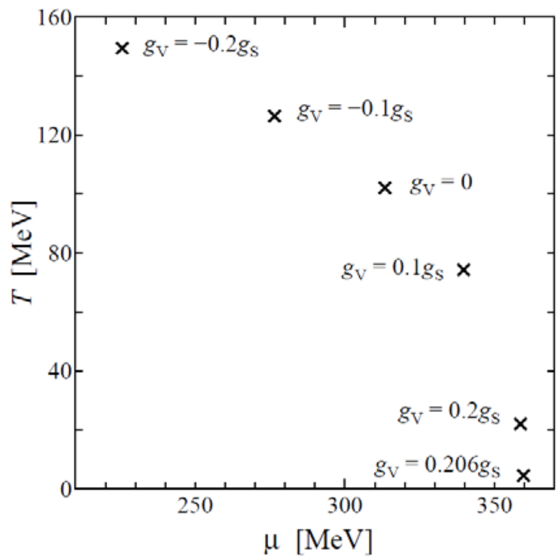
NJL and PNJL model with two or three flavor.



M. Kitazawa, T. Koide, T. Kunihiro and Y. Nemoto, Prog. Theor. Phys. **108** (2002) 929.



K. K., H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Lett. **B662** (2008) 26.



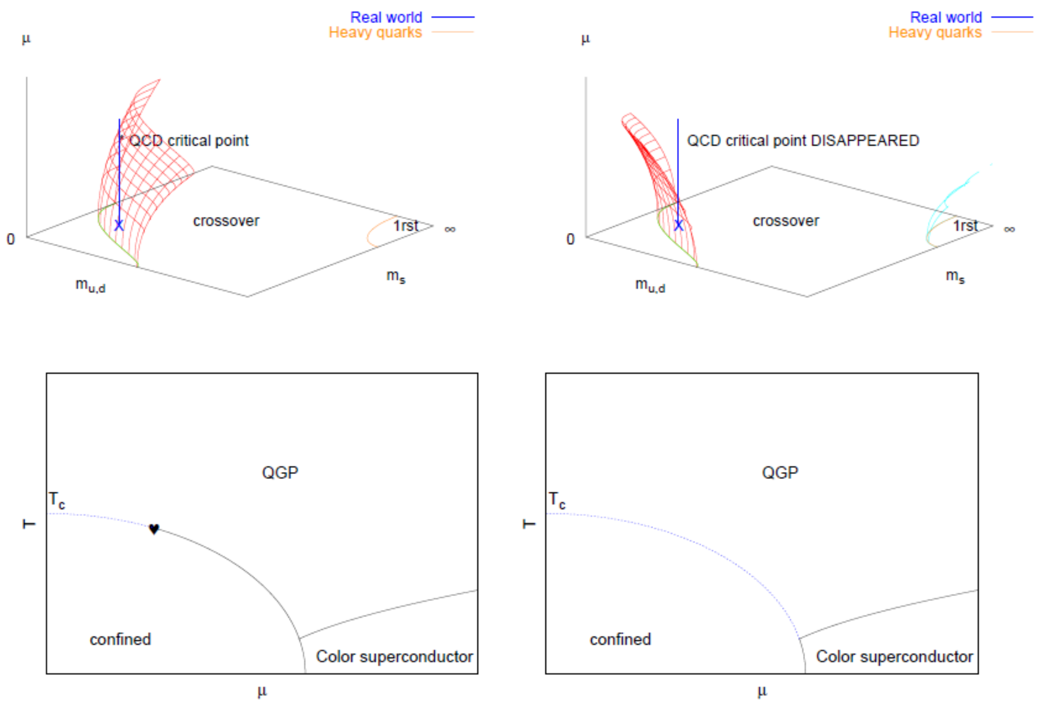
K. Fukushima, Phys. Rev. D **77** (2008) 114028.

Problem 2-1b

Critical endpoint

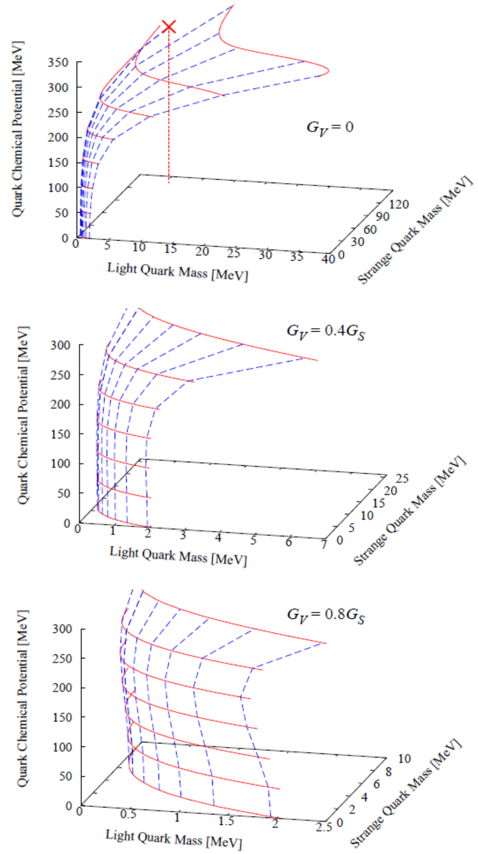
Is there the critical point on QCD phase diagram?

Obtained by LQCD with Taylor expansion



P. de Forcrand and O. Philipsen, JHEP **0701** (2007) 077;
P. de Forcrand, S. Kim and O. Philipsen, PoS LAT2007 (2007) 178;
P. de Forcrand and O. Philipsen, arXiv:hep-lat/0808.1096.

PNJL model



K. Fukushima, arXiv:hep-ph/08093080

Problem 2-1c

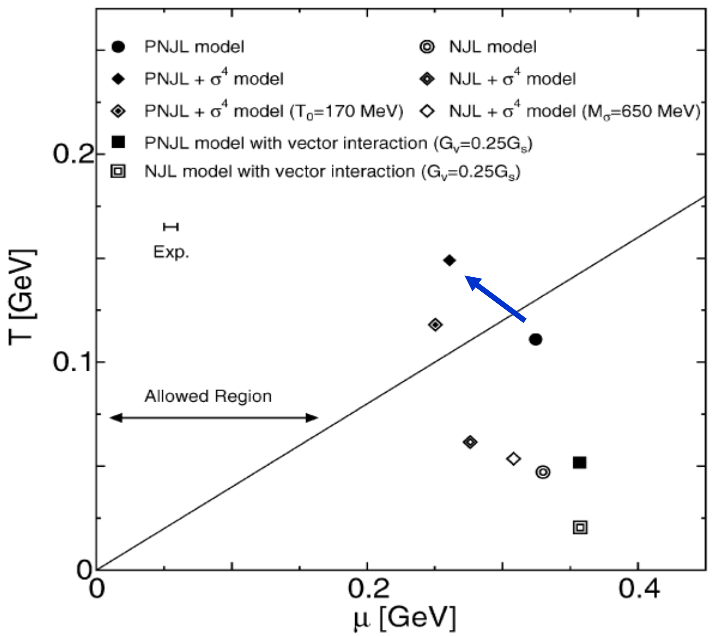
Is there critical endpoint ?

Ambiguity

$(\bar{q}q)^4$ type

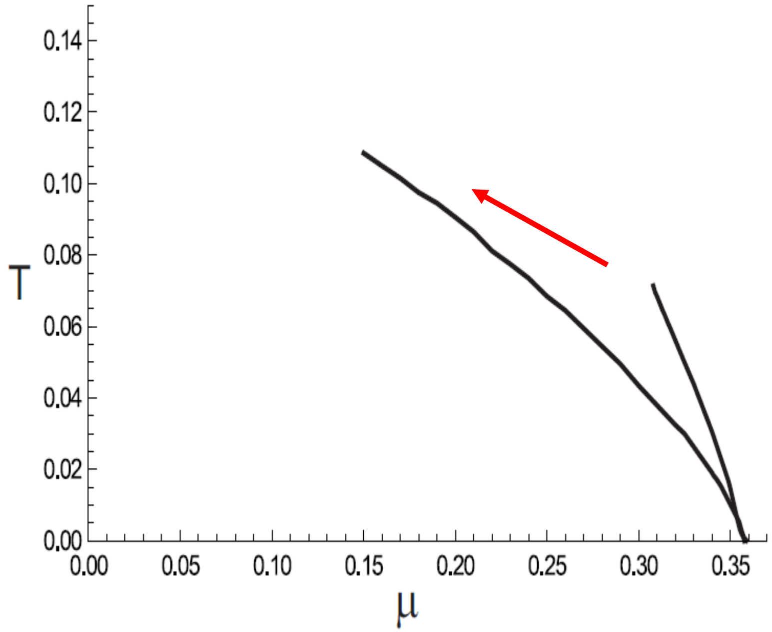
Effects to critical endpoint of the eight-quark interaction.

Two flavor



K. K., H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Lett. **B662** (2008) 26.

Three flavor



B. Hiller, J. Moreira, A. A. Osipov , A. H. Blin, arXiv:hep-ph/0812.1532.

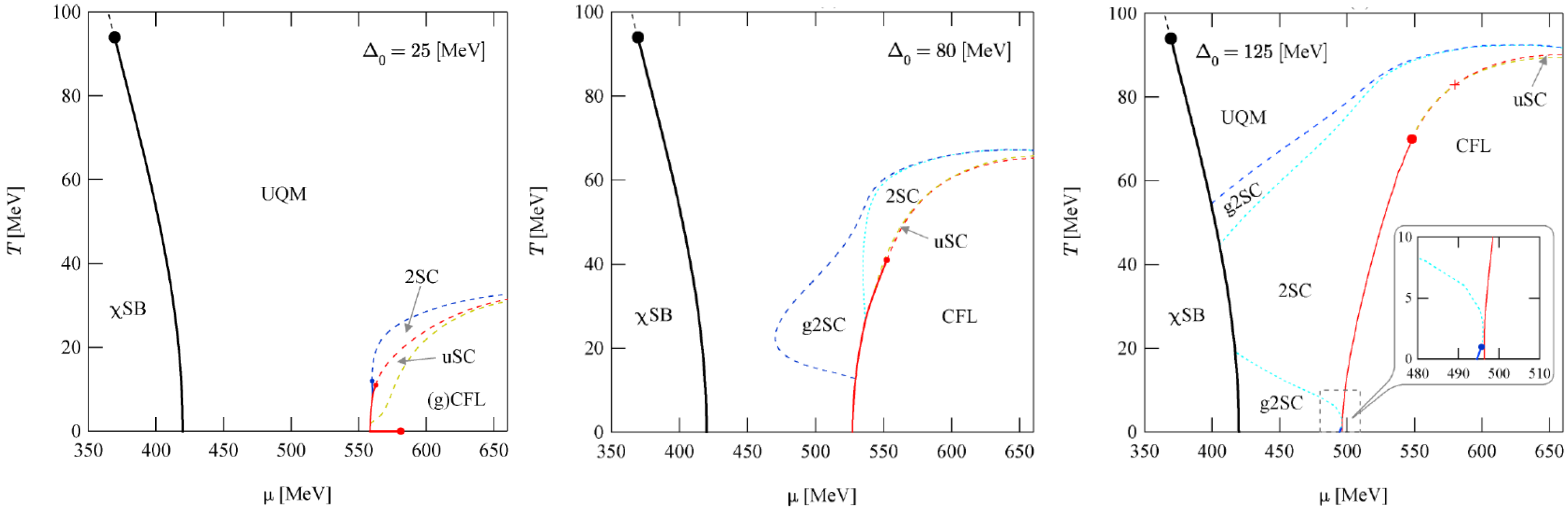
Problem 2-1d

Ambiguity

Phase structure at high density

Ex.) Nambu—Jona-Lasinio (NJL) model (H. Abuki and T. Kunihiro, Nucl. Phys. **A768** (2006) 118)

Phase diagram of three flavor system with diquark



The phase structure is modified due to **the strength** of the quark-quark interaction qualitatively.

Problem 2-1c

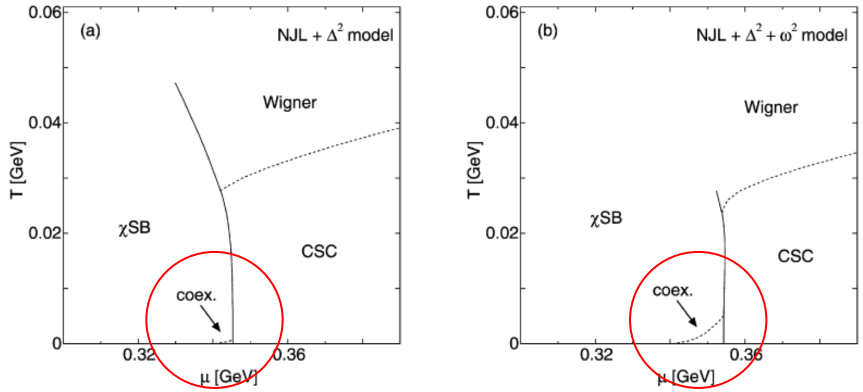
Ambiguity

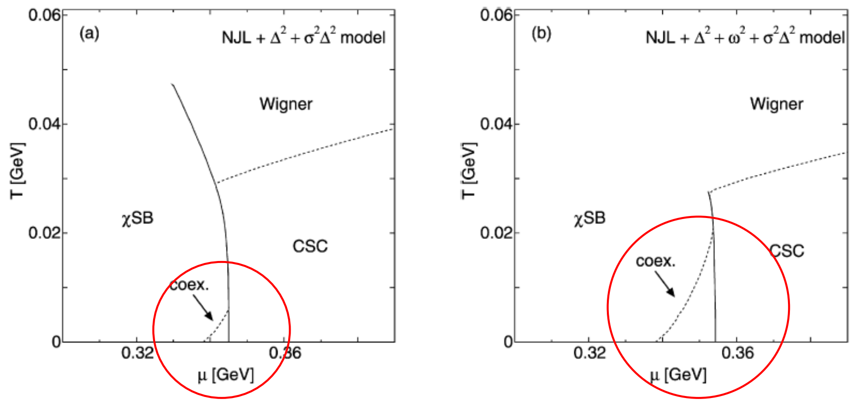
Effects of mixing interaction of chiral and diquark condensate

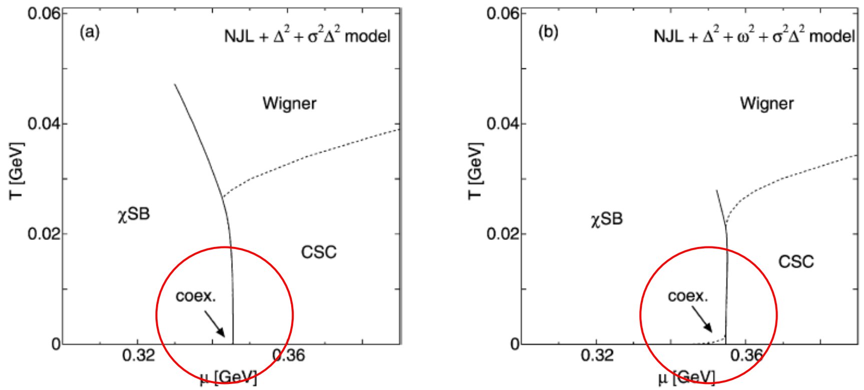
Phase structure is modified by **mixing interaction**.

ex.) Nambu—Jona-Lasinio (NJL) model

(K. K., M. Matsuzaki, H. Kouno, and M. Yahiro,
Phys. Lett. **B 657** (2007) 143)







The diquark and the chiral condensate mixing interaction affects the phase structure **near the transition line**.

Higher-order terms are important at finite temperature and chemical potential

Problem 2-2(

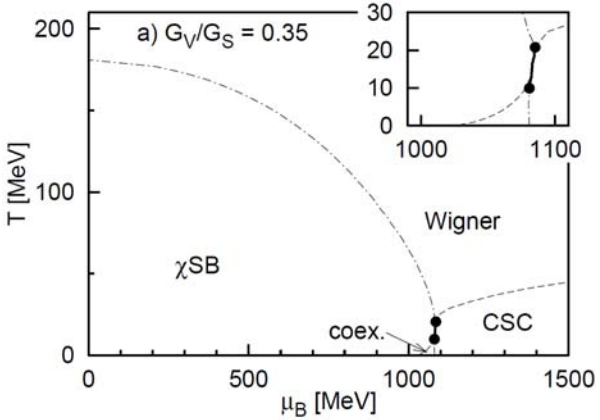
Critical endpoint

If the critical endpoint appears at QCD phase diagram, next serious problem arise.

How many critical points are there on QCD phase diagram?

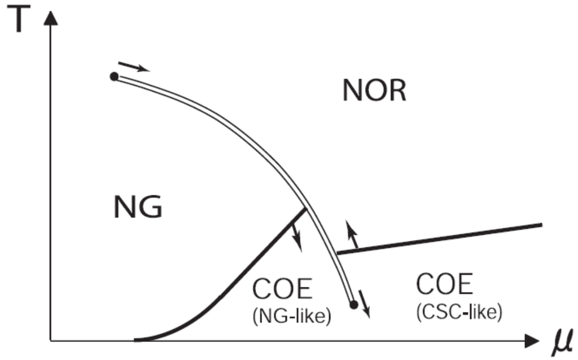
Investigation of critical endpoint by several approach.

NJL+CSC+G_v case



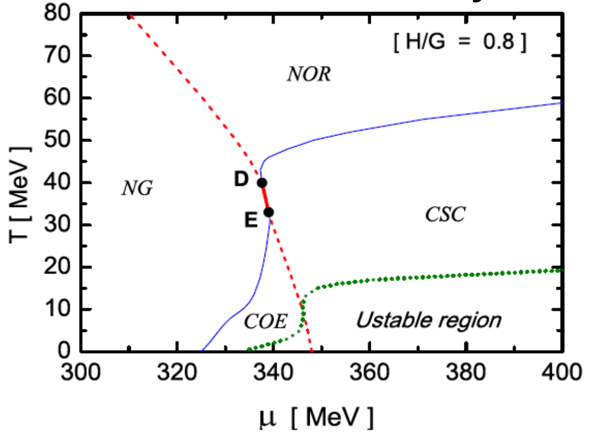
M. Kitazawa, T. Koide, T. Kunihiro and Y. Nemoto, Prog. Theor. Phys. **108** (2002) 929.

Ginzburg-Landau approach



T. Hatsuda, M. Tachibana, N. Yamamoto and G. Baym, Phys. Rev. lett, **97** (2006) 122001.

NJL+CSC+neutrality case

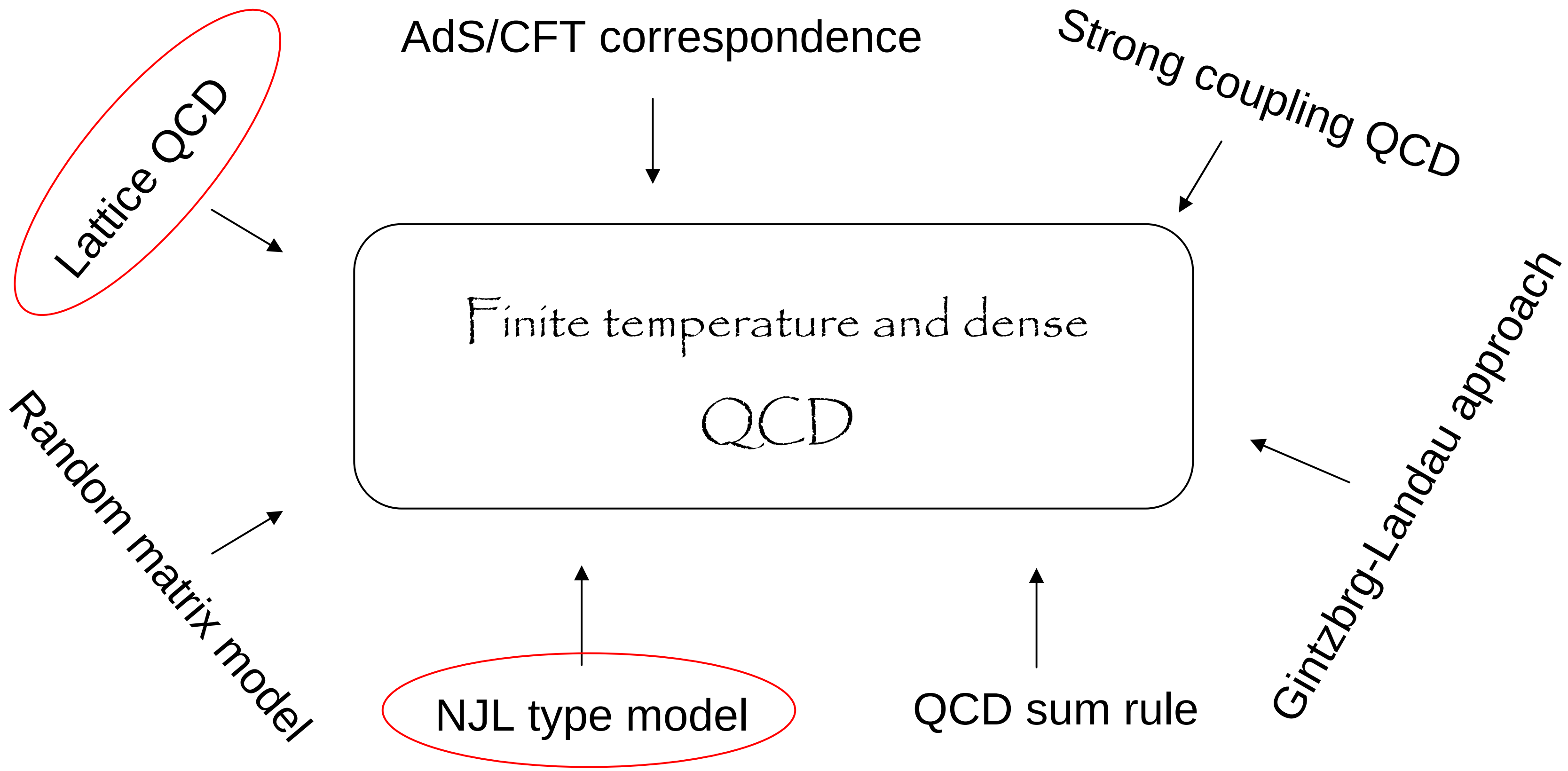


Z. Zhang, K. Fukushima and T. Kunihiro, arXiv:hep-ph/0808.3371.

Virtually the structure of QCD is **not clear** at high and moderate chemical potential .

Problem

LQCD, limited version of QCD and effective models have week point, respectively.



General knowledge obtained by several approach is important

Our approach

Our purpose

We **can not** obtain the exact QCD phase diagram by using **LQCD** or **effective models**, respectively.

It suggest that

“**an approach combining LQCD and other approach are necessary**”.

(In our study, we use the NJL type model)

Our main purpose

To obtain the exact QCD phase diagram

Point

Imaginary to real chemical potential

We pay attention to **imaginary chemical potential**.

In the imaginary chemical potential,
we **can exactly** calculate **LQCD**.

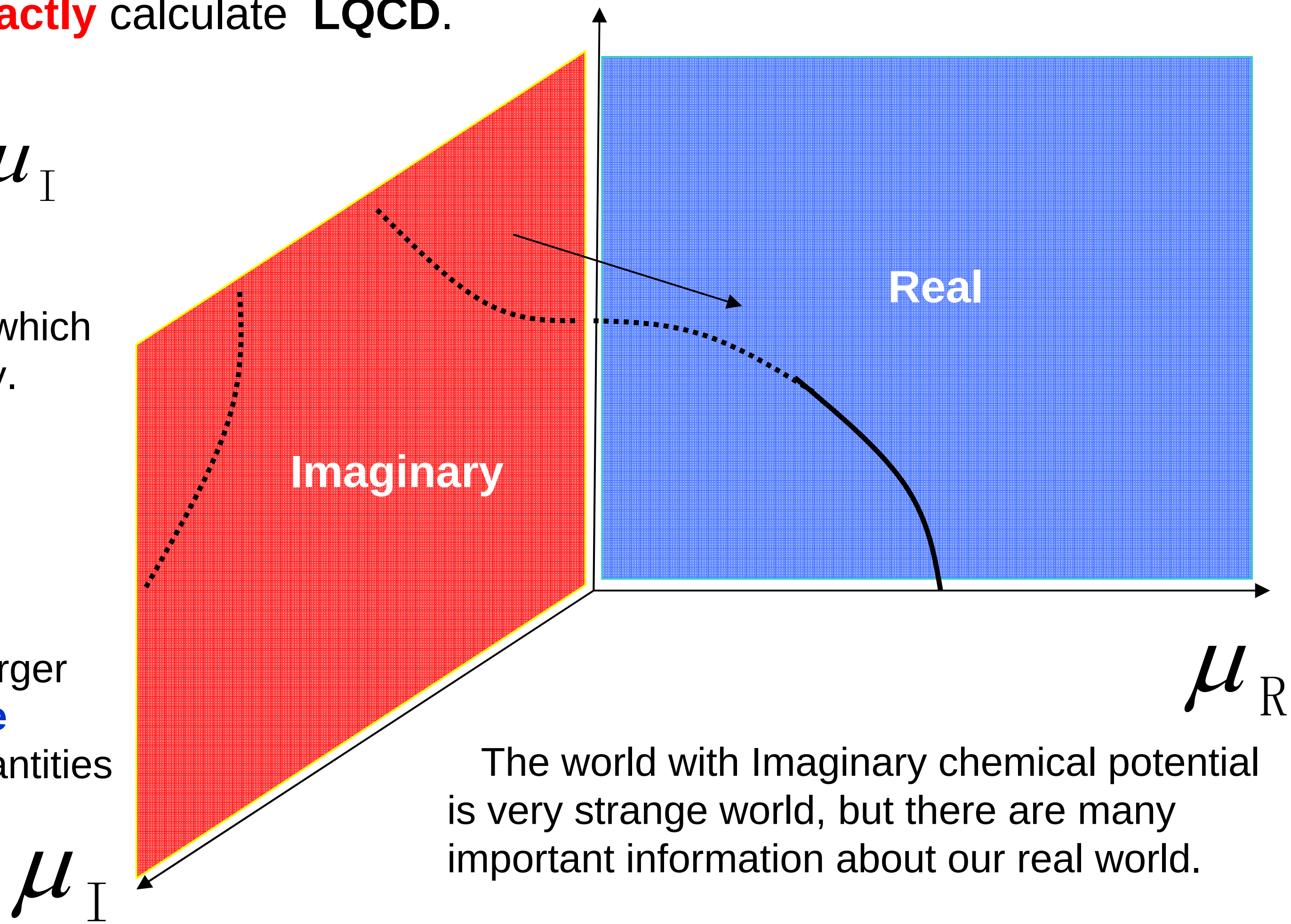
$\mu_R \longrightarrow \mu_I$

Some works are done which are based on LQCD only.

↑

The **reliability is lost** if μ_R become larger and larger because they **assume the fitting function** about quantities by LQCD data at μ_I.

Close relations between real and imaginary chemical potential!



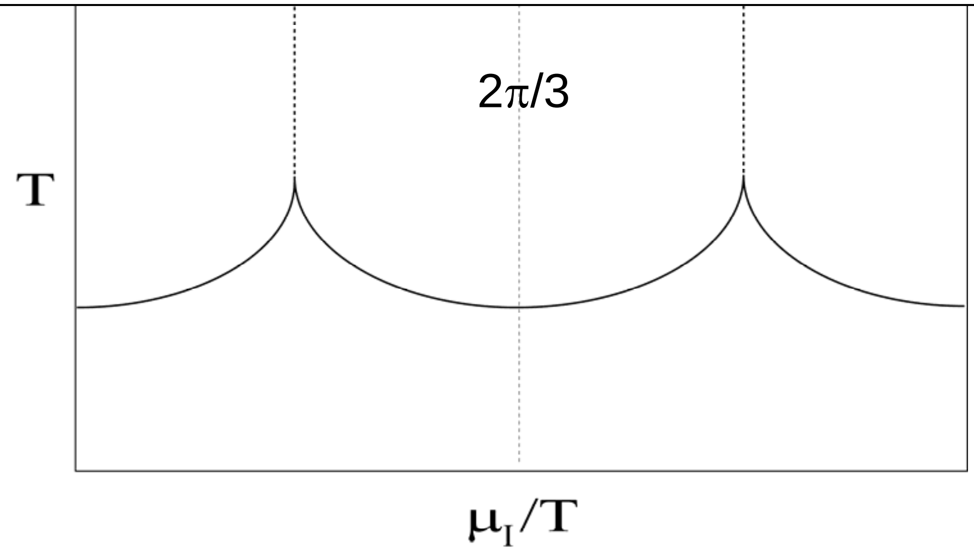
The world with Imaginary chemical potential is very strange world, but there are many important information about our real world.

Imaginary chemical potential matching approach

Strange world

Imaginary chemical potential

Phase diagram at imaginary chemical potential



2π/3 periodicity

Partition function of SU(N) gauged theory with imaginary chemical potential

$$Z(\theta) = \int D\psi D\bar{\psi} DA_{\mu} \ \exp\left[-\int d^4x \ \left\{ \bar{\psi}(\gamma D - m_0)\psi - \frac{1}{4}(F_{\mu\nu})^2 - i\frac{\theta}{\beta}\bar{\psi}\gamma_4\psi \right\} \right]$$

$$\mu = \frac{\theta}{\beta}, \quad D^{\nu} = \partial^{\nu} + iA^{\nu}, \quad A^{\nu} = gA_a^{\nu}\frac{\lambda^a}{2}$$

λ_a is the Gell-Mann matrix, g is the gauge coupling constant

Strange world

Imaginary chemical potential

$$\psi(x,\tau) \rightarrow \exp(i\tau\theta/\beta)\psi(x,\tau)$$

↓

$$Z(\theta) = \int D\psi D\bar{\psi} DA_\mu \ \exp\left[-\int d^4x \ \left\{\bar{\psi}(\gamma D - m_0)\psi - \frac{1}{4}(F_{\mu\nu})^2\right\}\right]$$

↓ Boundary condition : $\psi(x,\beta) = \exp(i\theta)\psi(x,0)$

(Gauge)

Z_N transformation $\quad \psi = U\psi \ ; \quad A \rightarrow UAU^{-1} - \frac{i}{g}(\partial U)U^{-1} \quad \left(U(x,\beta) = \exp\left(\frac{2\pi ik}{N}\right) U(x,0) \right)$

Boundary condition : $\psi(x,\beta) = \exp(2\pi ik/N)\exp(i\theta)\psi(x,0)$

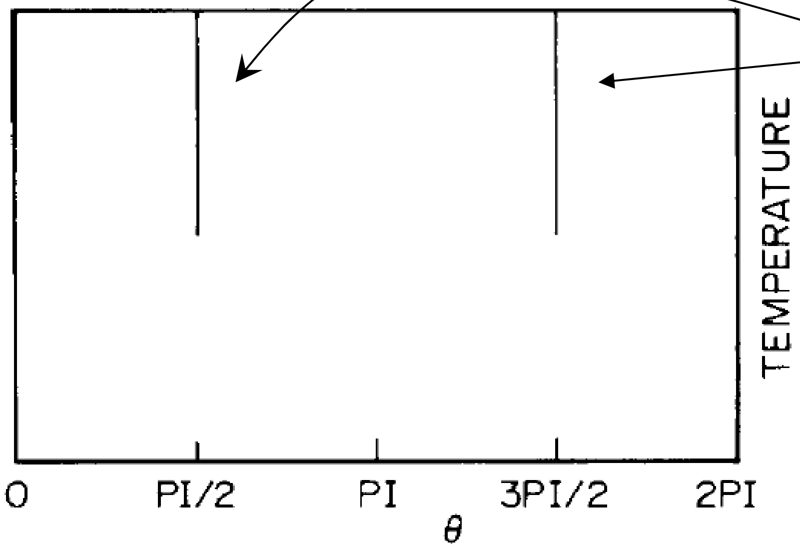
$$Z(\theta) = Z\left(\theta + \frac{2\pi k}{N}\right)$$

Z（θ）has the periodicity of 2π k/N !!

Strange world

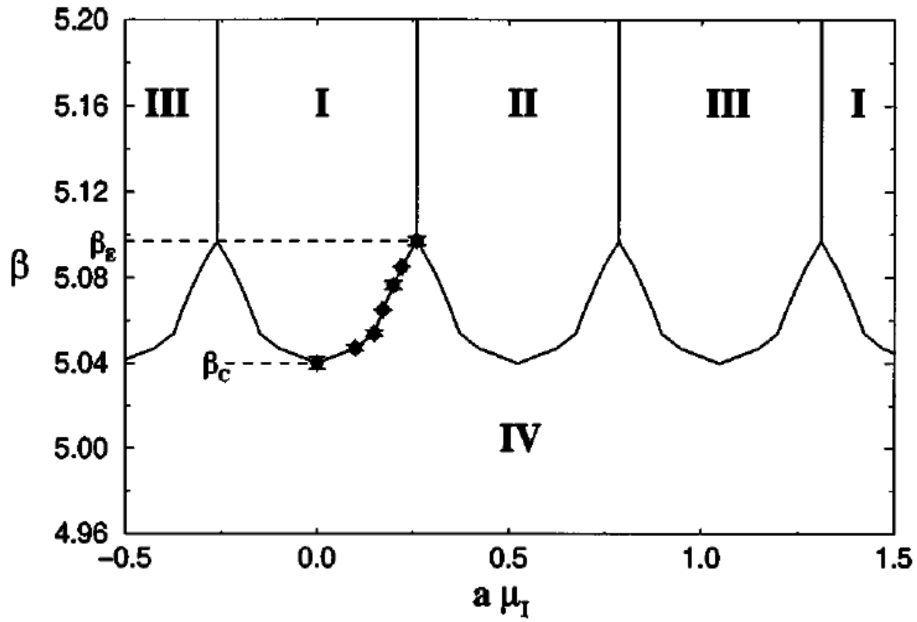
Imaginary chemical potential

It is called the “**Roberge-Weiss transition**”



A. Roberge and N. Weiss,
Nucl. Phys. **B** 275 (1986) 735

(From LQCD)



M. D’Elia, Phys. Rev. **D** 67 (2003) 014505.

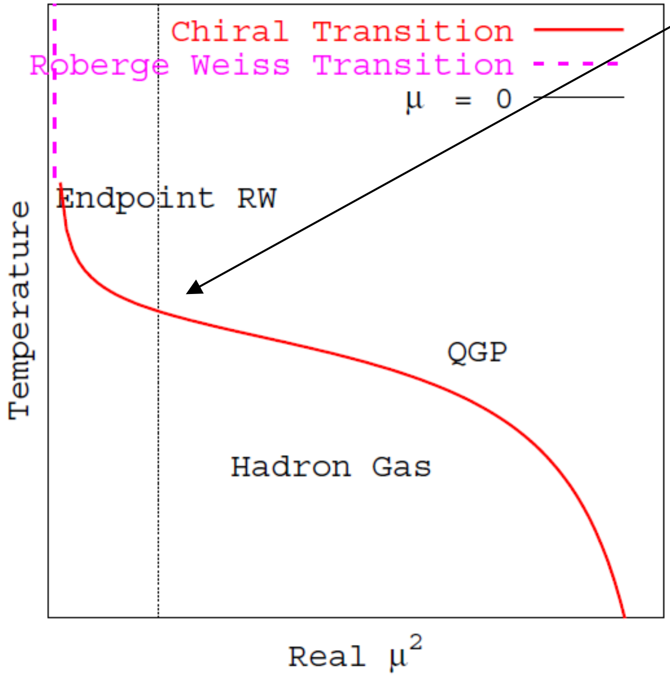
RW periodicity is reproduced
by using LQCD calculation.

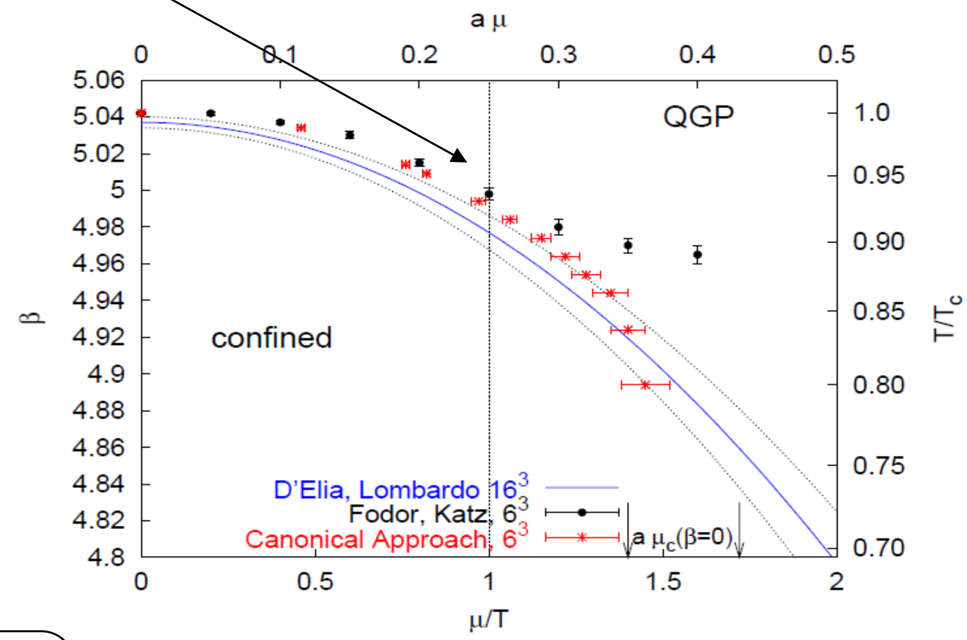
Strange world

Imaginary chemical potential

(4-flavor)

We assume the smooth connection at μ^2=0.





M. P. Lombardo, PoSCPOD2006 (2006) 003, hep-lat/0612017.

$$P[N,M](\mu_i) = \frac{a_0 + a_1\mu_i + \cdots + a_M \mu_i^M}{b_0 + b_1\mu_i + \cdots + b_N \mu_i^N}$$

P[0,M] is corresponding the to Taylor N-th partial sum.

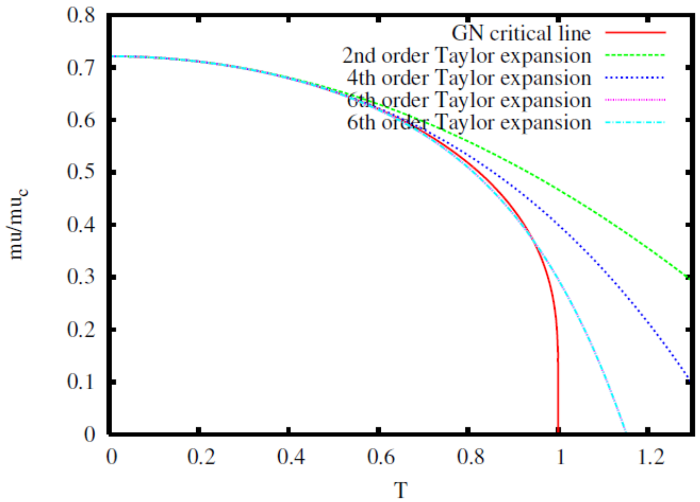
Those results suggest that the physics at real μ is **deeply related** to imaginary one.
This strange world is **useful** space because there are many important information about real chemical potential.

Extrapolation

Good method?

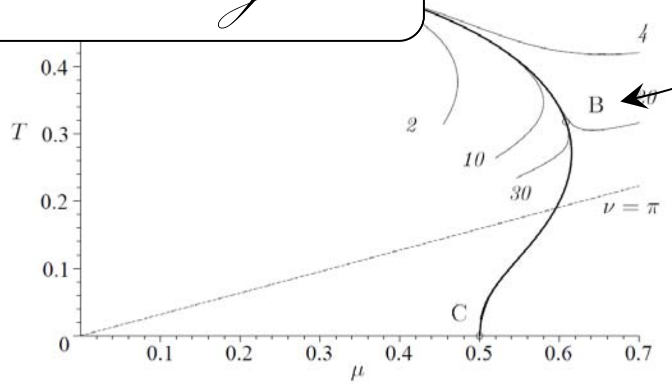
ex.) Gross-Neveu model

More simpler model than the NJL (QCD motivated) model



Phase diagram

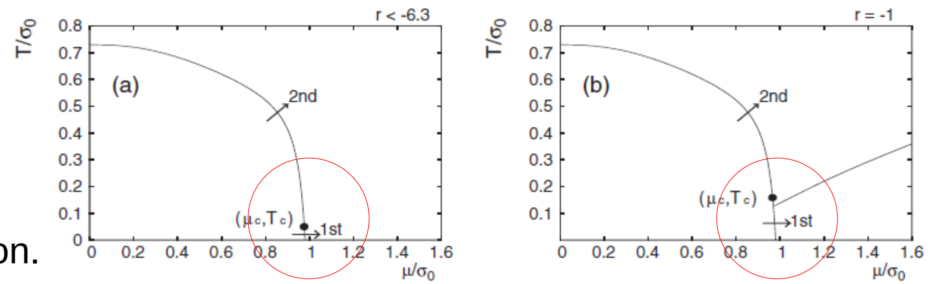
Tri-critical point



F. Karbstein and M. Thies, Phys. Rev. D 75 (2007) 025003.
M. P. Lombardo, PoSCPOD2006 (2006) 003, hep-lat/0612017.

But,

if the CSC is take into account,
the phase structure is modified.



In this **simple** case, we **can not** reach the structure at high chemical potential region by using the extrapolation.

H. Kohyama, Phys. Rev. D **77** (2008) 045016.

Numerical extrapolation

In QCD

Point of extrapolation

Thermodynamical value become **θ-even** or **θ-odd** function at imaginary chemical potential.

ex.)

θ-even ･･･: chiral condensate, real part of modified Polyakov-loop

θ-odd: ･･･ quark number density,
imaginary part of modified Polyakov-loop

CP invariance ⟶ Quantities are function of μ^2.

Relation between imaginary and real chemical potential

Fourier representation: $$Z_{\text{Canonical}}(T,B)=\int_{-\infty}^{+\infty} d\left(\frac{\mu_I}{T}\right) e^{-iB\mu_I/T} Z_{\text{Grand Canonical}}$$

Fugacity expansion: $$Z_{\text{Grand Canonical}}(T,\mu_R)=\sum_{B=-\infty}^{+\infty} e^{B\mu_R/T} Z_{\text{Canonical}}(T,B)$$

Truncation of expansion

In QCD

We can fit the θ-even function at imaginary chemical potential by using following function.

(But the truncation **can be proved** in simple case, but it **can not be proved** in realistic QCD)

Taylor expansion (P[0,M])

$$P[N,M](\mu_i) = \frac{a_0 + a_1\mu_i + \cdots + a_M \mu_i^M}{b_0 + b_1\mu_i + \cdots + b_N \mu_i^N}$$

P[0,M] is corresponding the to Taylor N-th partial sum.

ex.)

P. de Forcrand, O. Philipsen,
Nucl. Phys. B **642** (2002) 290.

P. de Forcrand, O. Philipsen,
Nucl. Phys. B **673** (2003) 170.

(~4-th order)

$$F(\theta) = \sum_k a_k \cos 3k\theta \longrightarrow F(\mu_R/T) = \sum_k a_k \cosh(3k\mu_R/T)$$

(Anytime, **k=0** is imposed)

Therefore,

$$F(\mu) = A(T)\frac{e^{3i\theta} + e^{-3i\theta}}{2} + C(T)$$

$$\begin{cases} A(T) = \dfrac{F(T,\theta=0) - F(T,\theta=\pi/3)}{2} \\ C(T) = \dfrac{F(T,\theta=0) + F(T,\theta=\pi/3)}{2} \end{cases}$$

ex.)

M. D'Elia and M. P. Lombardo,
Phys. Rev. D **67** (2003) 014505.

M. D'Elia and M. P. Lombardo,
Phys. Rev. D **70** (2004) 074509.

Our approach

Imaginary chemical potential

An approach which only based on LQCD has **limit** !

- Error of LQCD simulation
- Choice of fitting function.
  - Order of fitting
  - Truncation

Therefore, we **extend** the effective model at **imaginary chemical potential**.

We call this approach

"Imaginary chemical potential matching approach".

Formalism

PNJL model

We introduce

$G_{s8}\left((\bar{q}q)^2+(\bar{q}i\gamma_5\vec{\tau}q)^2\right)^2$ and $G_v\left(\bar{q}\gamma^\mu q\right)^2$

in below Lagrangian density

at actual calculation.

The Lagrangian density of the NJL model

$$\mathcal{L}=\bar{q}(i\gamma^\mu\partial_\mu-m_0)q+G_s\left((\bar{q}q)^2+(\bar{q}i\gamma_5\vec{\tau}q)^2\right)$$

- Spontaneous chiral symmetry braking

The Lagrangian density of the PNJL model

K.Fukushima, Phys. Lett. **B591** (2004) 277.

$$\mathcal{L}=\bar{q}(i\gamma^\mu D_\mu-m_0)q+G_s\left((\bar{q}q)^2+(\bar{q}i\gamma_5\vec{\tau}q)^2\right)-U(\bar{\Phi},\Phi)$$

- Spontaneous chiral symmetry braking
- (Approximated) confinement mechanism

The PNJL model **can reproduce** the LQCD data at μ=0 very well.

PNJL model

Importance of multi-leg interaction

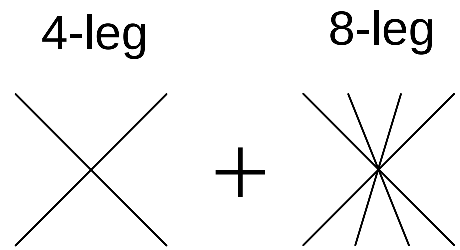
K. K., H. Kouno, T. Sakaguchi, M. Matsuzaki and M. Yahiro, Phys. Lett. **B647** (2007) 446.

K. K., M. Matsuzaki, H. Kouno, and M. Yahiro, Phys. Lett. **B657** (2007) 143.

K. K., H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Lett. **B662** (2008) 26.

Interactions of the NJL model ･･･ four quark interactions

(Scalar and pseudo-scalar type)



Usually, **other interactions are neglected** in the NJL model.
(vector type and higher-order multi-quark interaction)

But, there are **no reason** that neglect multi-quark interactions and the vector interaction in NJL model.

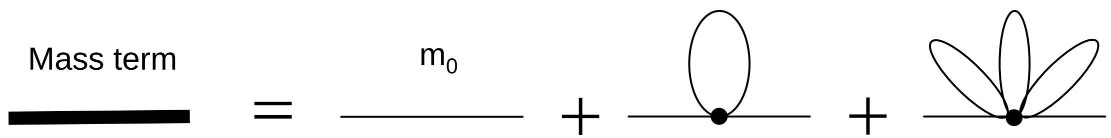
References of eight-quark interactions

A. A. Osipov, B.Hiller and J. da Providencia, Phys. Lett. B**634** (2006) 48.

A. A. Osipov, B.Hiller, J. Moreira and J. da Providencia, Phys. Lett. B**646** (2007) 91.

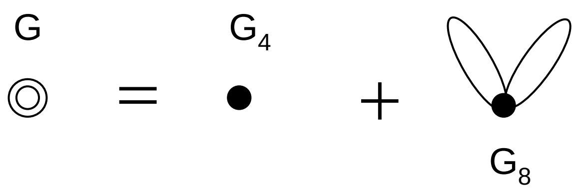
R. Huguet, J. C. Caillon and J. Labarsouque, Nucl. Phys. A**781** (2007) 448.

R. Huguet, J. C. Caillon and J. Labarsouque, Phys. Rev. C **75** (2007) 048201.



Same order (1/Nc expansion)

Eight quark interactions import the **temperature** and **chemical potential** dependence to coupling constants.



It is important for the vector interaction to consist with the strength at low and high chemical potential.

PNJL model

Thermodynamical potential

Parameters G_s and Λ, are fitted by using π meson mass and decay constant at T=μ=0

$$\frac{\Omega}{V} = U_P + U_M - 2N_f \int \frac{d^3p}{(2\pi)^3}\Big[N_c E(p) + T\ln\left(1+(\Phi+\bar{\Phi}e^{-\beta E^-})e^{-\beta E^-} + e^{-3\beta E^-}\right)$$

$$+T\ln\left(1+(\Phi+\bar{\Phi}e^{-\beta E^+})e^{-\beta E^+} + e^{-3\beta E^+}\right)\Big]$$

$$U_M = G_s\sigma^2 - G_v\omega^2$$

$$\left(m_0 = 5.5\text{ MeV} \right)$$

$$E^{\pm} = E \pm i\theta/\beta, \quad E = \sqrt{p^2+M^2}, \quad M = m_0 + 2G_s\sigma$$

Polynomial: $$\frac{U(\Phi,\bar{\Phi};T)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3+\bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

Logarithm: $$\frac{U(\Phi,\bar{\Phi};T)}{T^4} = -\frac{a(T)}{2}\bar{\Phi}\Phi - b(T)\ln\left(1-6\bar{\Phi}\Phi + 4(\Phi^3+\bar{\Phi}^3) + 3(\bar{\Phi}\Phi)^2\right)$$

$$b(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2$$

a_0	a_1	a_2	b_3
3.51	–2.47	15.2	–1.75

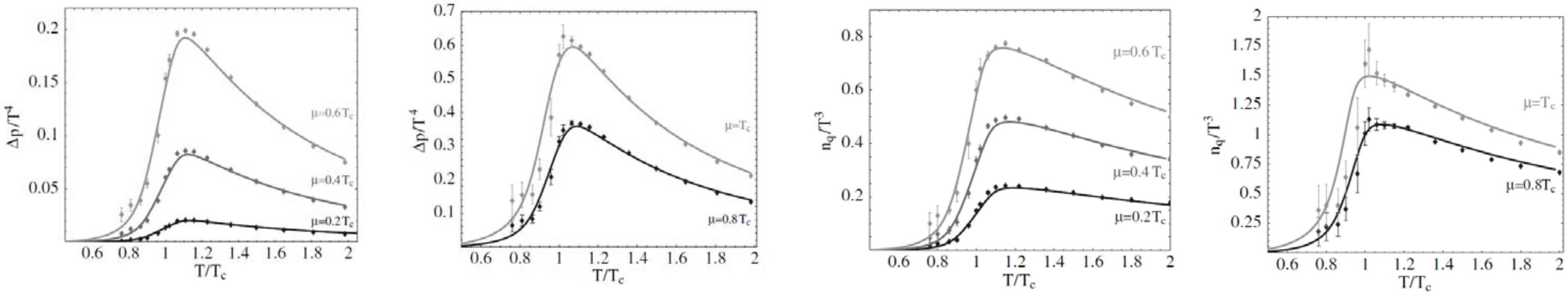
C. Ratti, et al. Eur. Phys. J. C **49** (2007) 213.

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

a_0	a_1	a_2	a_3	b_3	b_4
6.75	–1.95	2.625	–7.44	0.75	7.5

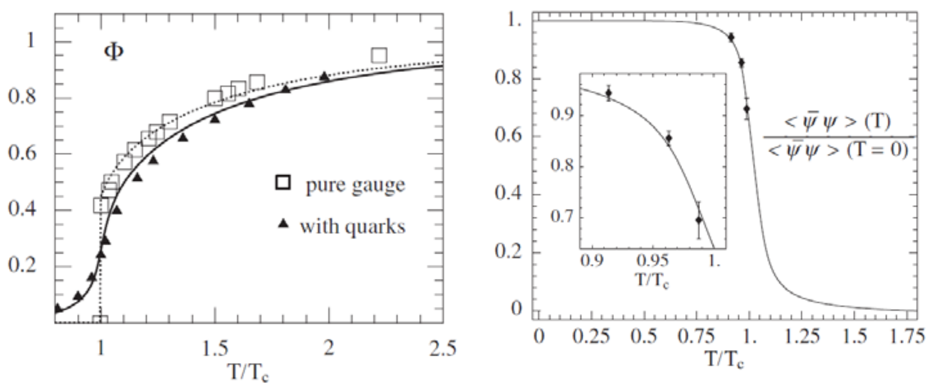
C. Ratti, et al. Phys. Rev. D**73** (2006) 014019.

Result of PNJL model at μ=0

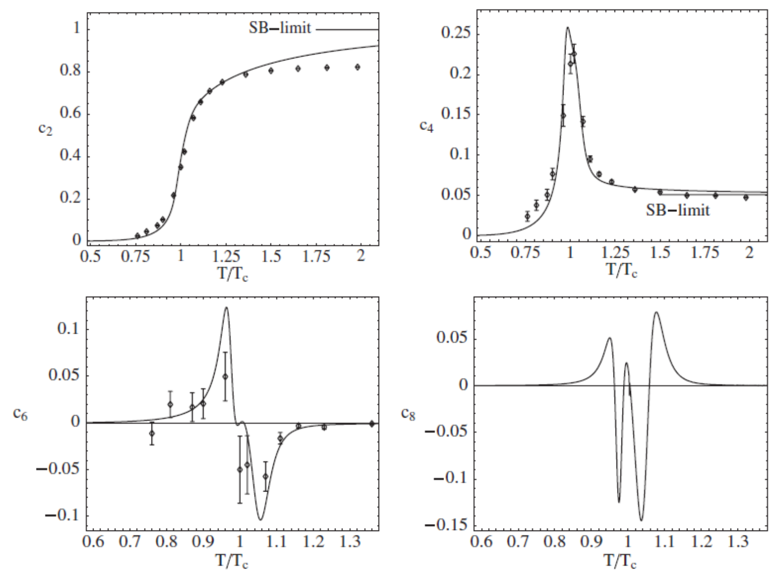


C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D **73** (2006) 014019.

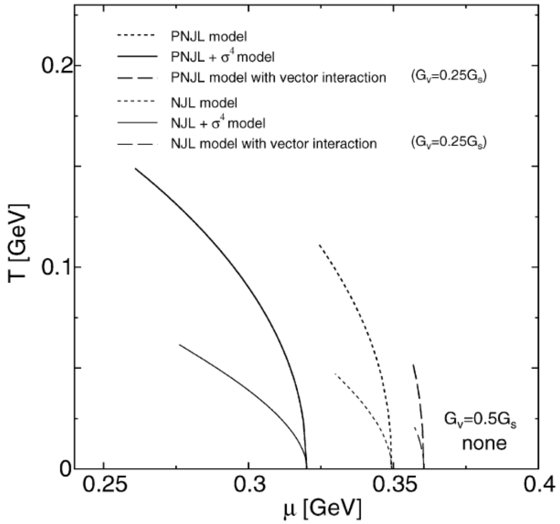
(Comparing with LQCD data C. R. Allton *et al.*, Phys. Rev. D **68**, (2003) 014507)



S. Ro¨ßner, C. Ratti, and W. Weise, Phys. Rev. D **75** (2007) 034007.

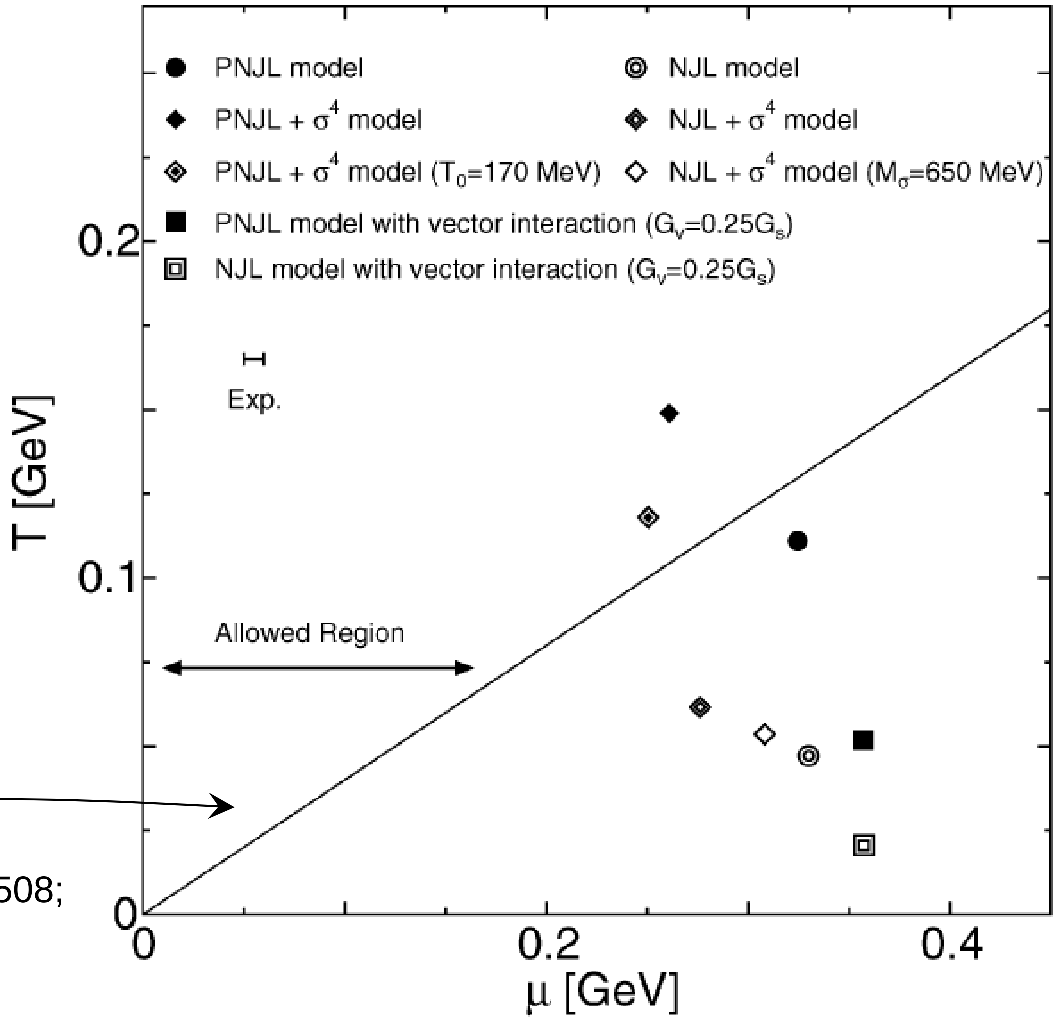


Result of PNJL model (Our result)



Vector interaction is essential!

K. K., H. Kouno, M. Matsuzaki, M. Yahiro, Phys. Lett. **B662** (2008) 26.



S. Ejiri, Phys. Rev. D **77** (2008) 014508; arXiv:hep-lat/0710.0653.

PNJL model

Symmetry

Thermodynamical potential of the usual PNJL model

$$\frac{\Omega}{V} = -2N_f \int \frac{d^3p}{(2\pi)^3} \Bigg[N_c E(p) + \frac{1}{\beta} \ln\left(1 + 3(\Phi + \bar{\Phi} e^{-\beta E^-(p)}) e^{-\beta E^-(p)} + e^{-3\beta E^-(p)}\right)$$
$$+ \frac{1}{\beta} \ln\left(1 + 3(\bar{\Phi} + \Phi e^{-\beta E^+(p)}) e^{-\beta E^+(p)} + e^{-3\beta E^+(p)}\right) \Bigg] + U_M + U_p$$

Z_3 transformation $\quad \Phi \to \Phi e^{-i2\pi k/3}, \quad \bar{\Phi} \to \bar{\Phi} e^{i2\pi k/3}$

The Polyakov Potential U is invariant, but the NJL part is not invariant.

Extended Z_3 transformation $\quad e^{\pm i\theta} \to e^{\pm i\theta} e^{\pm i2\pi k/3}, \; \Phi \to \Phi e^{-i2\pi k/3}, \quad \bar{\Phi} \to \bar{\Phi} e^{i2\pi k/3}$

The Polyakov Potential U is invariant, and the NJL part is also invariant.

PNJL model

Symmetry

Modified Polyakov-loop

We can easily see the RW periodicity by using the **modified Polyakov-loop**.

Therefore, we determine the **modified Polyakov-loop** like as

$$\Psi = e^{i\theta}\Phi, \quad \bar{\Psi} = e^{-i\theta}\bar{\Phi}.$$

Thermodynamical potential of the PNJL model with modified Polyakov-loop

$$\frac{\Omega}{V} = -2N_f \int \frac{d^3p}{(2\pi)^3}\left[N_c E(p) + \frac{1}{\beta}\ln\left(1 + 3\Psi e^{-\beta E} + 3\bar{\Psi} e^{-2\beta E} e^{\beta\mu_B} + e^{-3\beta E} e^{\beta\mu_B}\right)\right.$$

$$\left. + \frac{1}{\beta}\ln\left(1 + 3\bar{\Psi} e^{-\beta E} + 3\Psi e^{-2\beta E} e^{-\beta\mu_B} + e^{-3\beta E} e^{-\beta\mu_B}\right)\right] + U_M + U_p$$

PNJL model

Parameter set

K. K., H. Kouno, T. Sakaguchi, M. Matsuzaki and M. Yahiro, Phys. Lett. **B647** (2007) 446.
K. K., M. Matsuzaki, H. Kouno, and M. Yahiro, Phys. Lett. **B657** (2007) 143.
K. K., H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Lett. **B662** (2008) 26.

Parameters: Gs, Gv, **Gs8**

Our usual procedure

- Gs (scalar type 4-quark interaction)
- **Gs8** (scalar type 8-quark interaction)
- Λ (3 dimensional momentum cutoff)

⇐

$$M_\pi = 138\ MeV,\quad M_\sigma = 650\ MeV,\quad f_\pi = 93.3\ MeV$$

or

$$M_\pi = 138\ MeV,\quad M_\sigma = 600\ MeV,\quad f_\pi = 93.3\ MeV$$

G_v is **free parameter**.

Parameters: Gs, Gv, **Gs8**

Our new procedure

All parameters are fitted in **imaginary chemical potential region** (and μ=0 region).

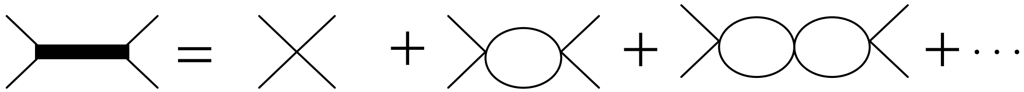
Y. Sakai, K. K, H. Kouno and M. Yahiro, Phys. Rev. D **77** (2008) 051901(R).
Y. Sakai, K. K, H. Kouno and M. Yahiro, Phys. Rev. D **78** (2008) 036001.
Y. Sakai, K. K, H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Rev. D **78** (2008) 076071.

PNJL model

Determination of meson mass

Results are shown in later

Random phase approximation



$$\begin{cases} 1-2G_s\Pi_{ss}=0 \\ 1-2G_s\Pi_{pp}=0 \end{cases}$$

$$\Gamma_{eff}^{(x)} = \Gamma_x \left[2iG_s + 2iG_s \left(\frac{1}{i}\Pi_x \right) 2iG_s + \cdots \right] \Gamma_x$$

$$= \Gamma_x \left(\frac{2iG_s}{1-2G_s\Pi_x} \right) \Gamma_x$$

$$\Pi_{ss} = -i\int \frac{d^4k}{(2\pi)^4} \frac{(p+q/2)-M^2}{[(p+q/2)^2-M^2+i\varepsilon]} \frac{1}{[(p-q/2)^2-M^2+i\varepsilon]}$$

$$\Pi_{pp} = -i\int \frac{d^4k}{(2\pi)^4} \frac{(p+q/2)+M^2}{[(p+q/2)^2-M^2+i\varepsilon]} \frac{1}{[(p-q/2)^2-M^2+i\varepsilon]}$$

At finite chemical potential in the PNJL model : $q_4 \rightarrow q_4 - (\mu - iA_4)$

H. Hansen, W. M. Alberico, A. Beraudo, A. Molinari, M. Nardi and C. Ratti, Phys. Rev. D **75** (2007) 065004.

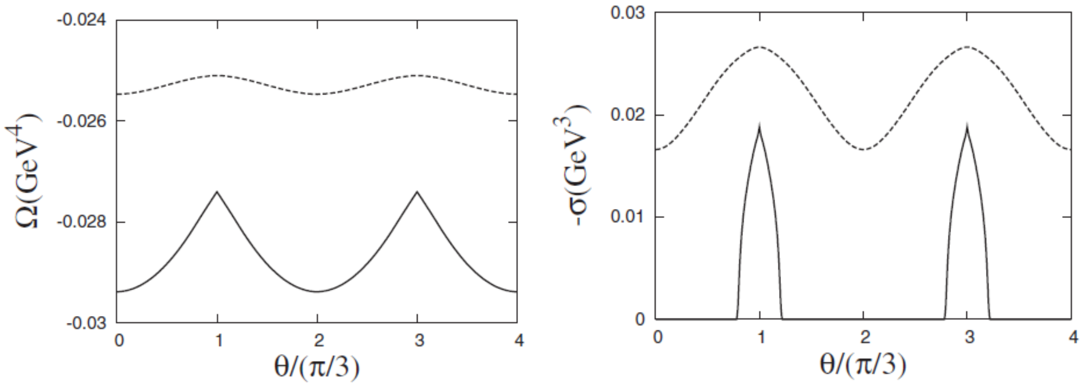
Result

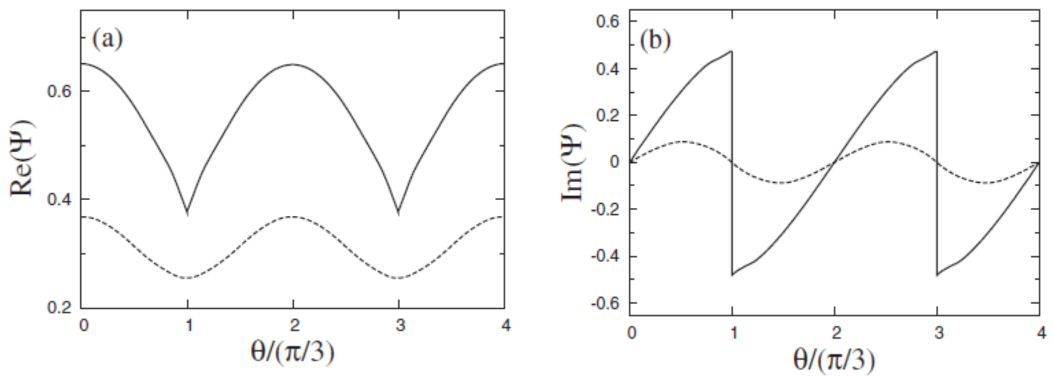
Our result 1-1

Y. Sakai, K. K, H. Kouno and M. Yahiro, Phys. Rev. D **77** (2008) 051901(R).

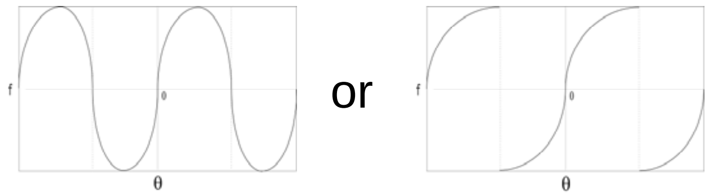
We show that the PNJL model can **qualitatively reproduce the RW periodicity**.

In chiral limit



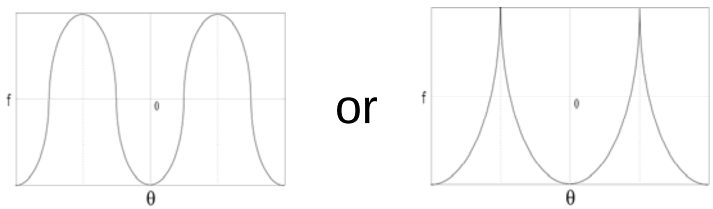


$$f(-\theta) = -f(\theta)$$



In the θ-odd case, the discontinuity **can appear**.

$$f(-\theta) = f(\theta)$$



In the θ-even case, the discontinuity **can not appear**.

Detail discussions of the propagation between order parameters are shown in K.K., Y. Sakai, H. Kouno, M. Matsuzaki and M. Yahiro, arXiv:hep-ph/0804.3557.

Our result 1-2

Many theory and effective models **do not have** RW periodicity,
but the PNJL model **has** the periodicity.

It is suggested that the PNJL model is **good model**
and has the possibility of well reproducing the various QCD prosperity.

ex.)

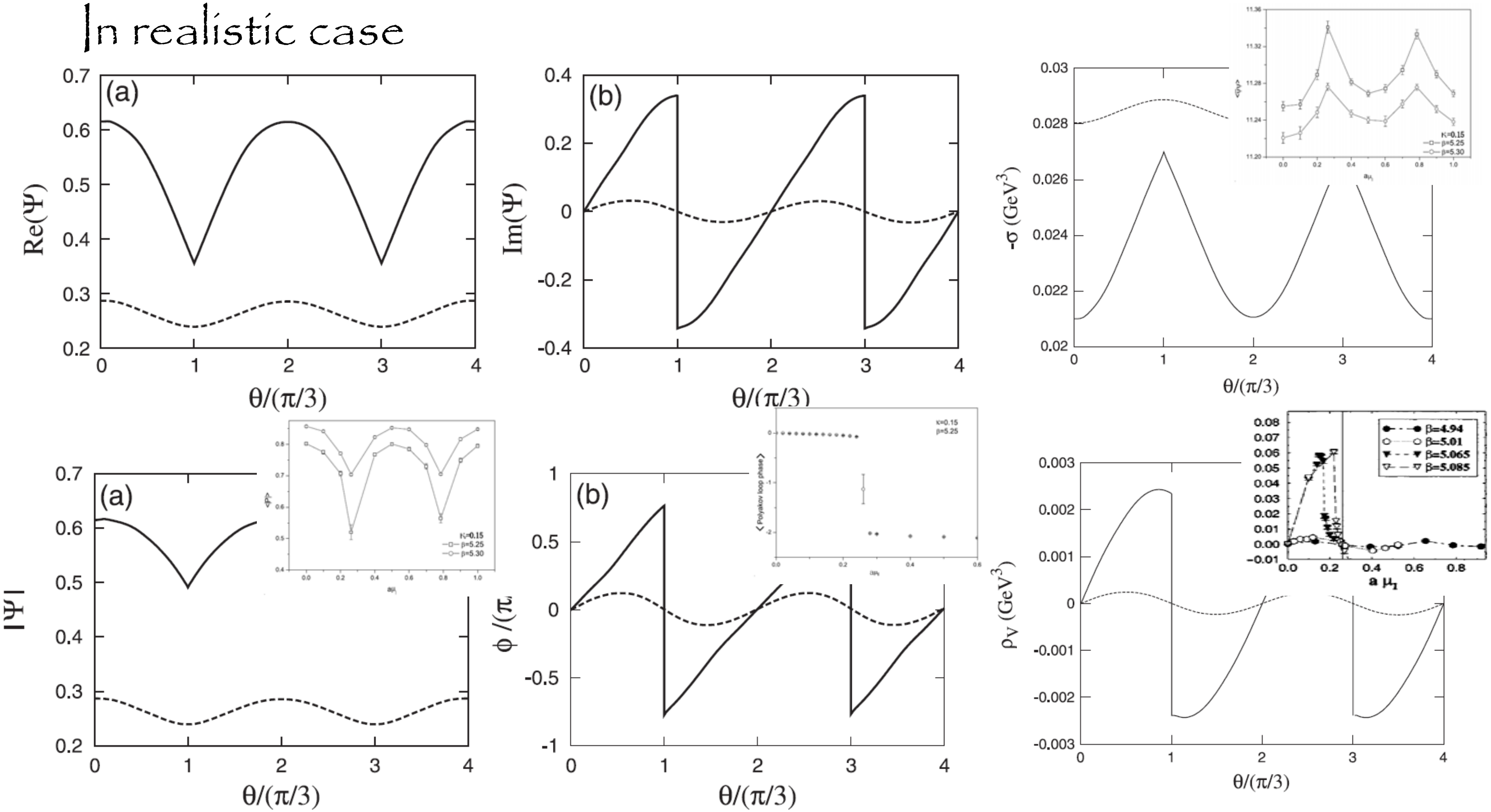
	Chiral	Z_3 (Pure gauge)	Extended Z_3	RW	
LQCD	○	○	○	○	Good
Potts model	×	○	○	○	Heavy quark mass limit are used.
Chiral random matrix model	○	×	×	×	In the recent theory, it has the possibility of reproducing the RW periodicity.
NJL model	○		×	×	There are no gluon degree of freedom explicitly.
PNJL model	○	○	○	○	Good

Our result 1-3

Y. Sakai, K. K, H. Kouno and M. Yahiro, Phys. Rev. D **78** (2008) 036001.

We show that the PNJL model can **qualitatively reproduce the RW periodicity**.

In realistic case



Lattice data: H. S. Chen and X. Q. Luo, Phys. Rev. D **72** (2005) 034504

M. D' Elia and M. P. Lambard, Phys. Rev. D **67** (2003) 145005.

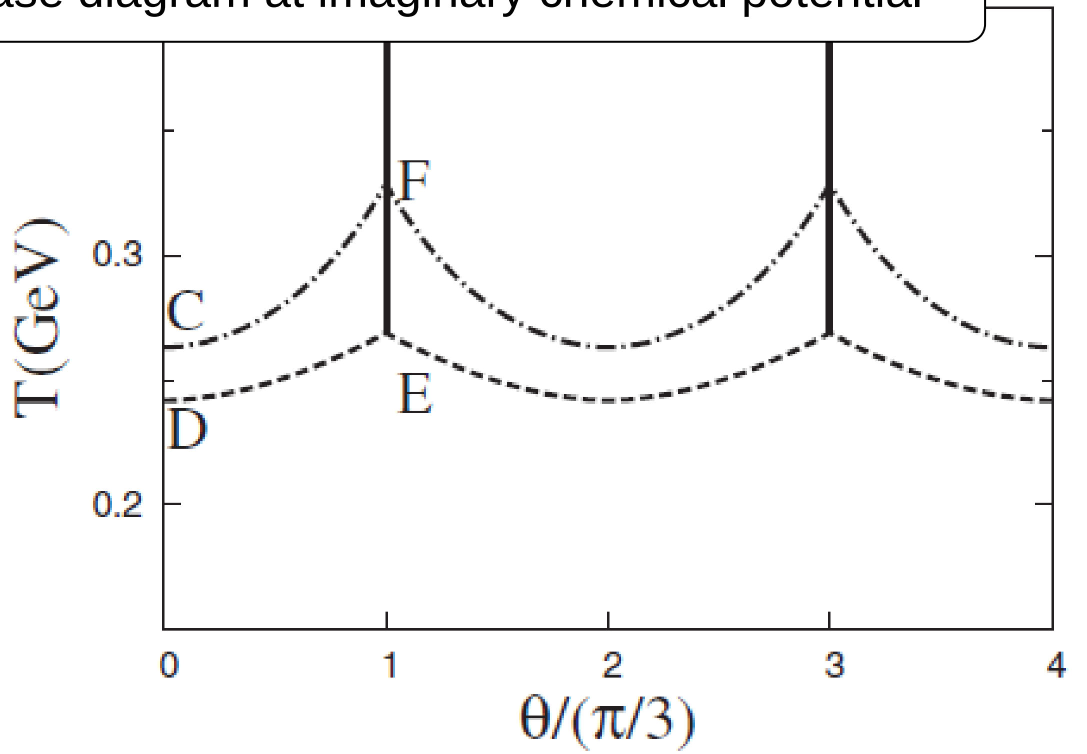
Our result 1-2

Y. Sakai, K. K, H. Kouno and M. Yahiro, Phys. Rev. D **78** (2008) 036001.

We show that the PNJL model can **qualitatively reproduce the RW periodicity**.

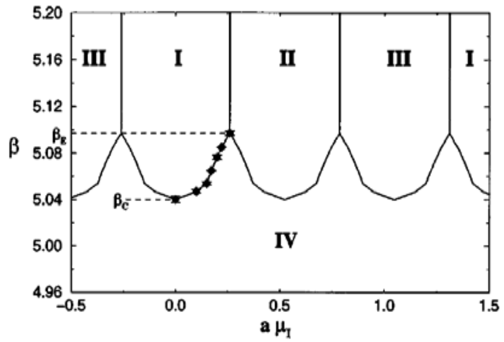
Phase diagram at imaginary chemical potential

Our result



Good correspondence!

LQCD

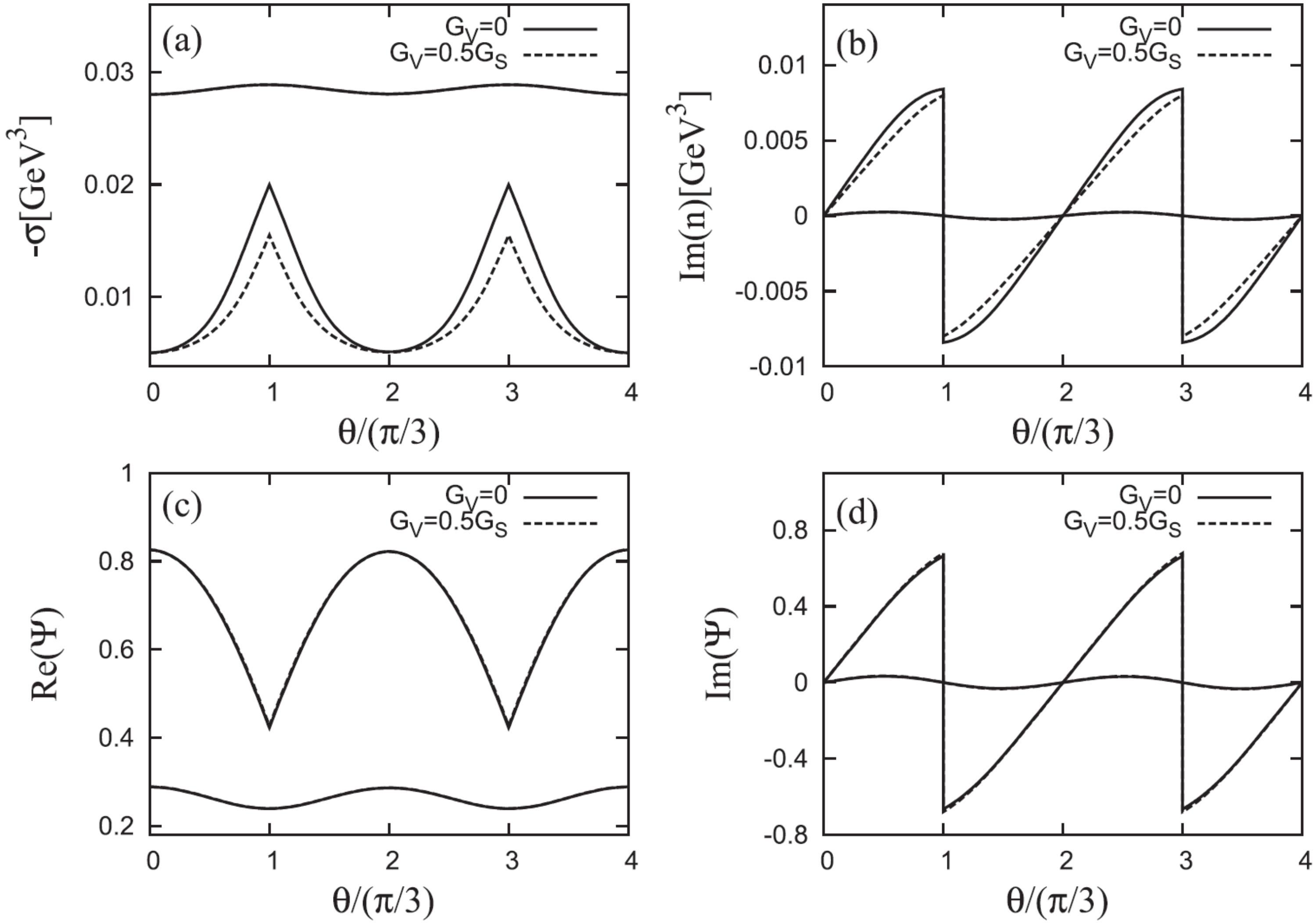


Therefore, the PNJL model **good model** !!

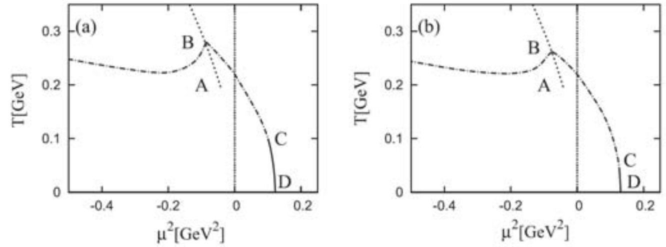
Our result 2-1

Y. Sakai, K. K, H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Rev. D **78** (2008) 076071.

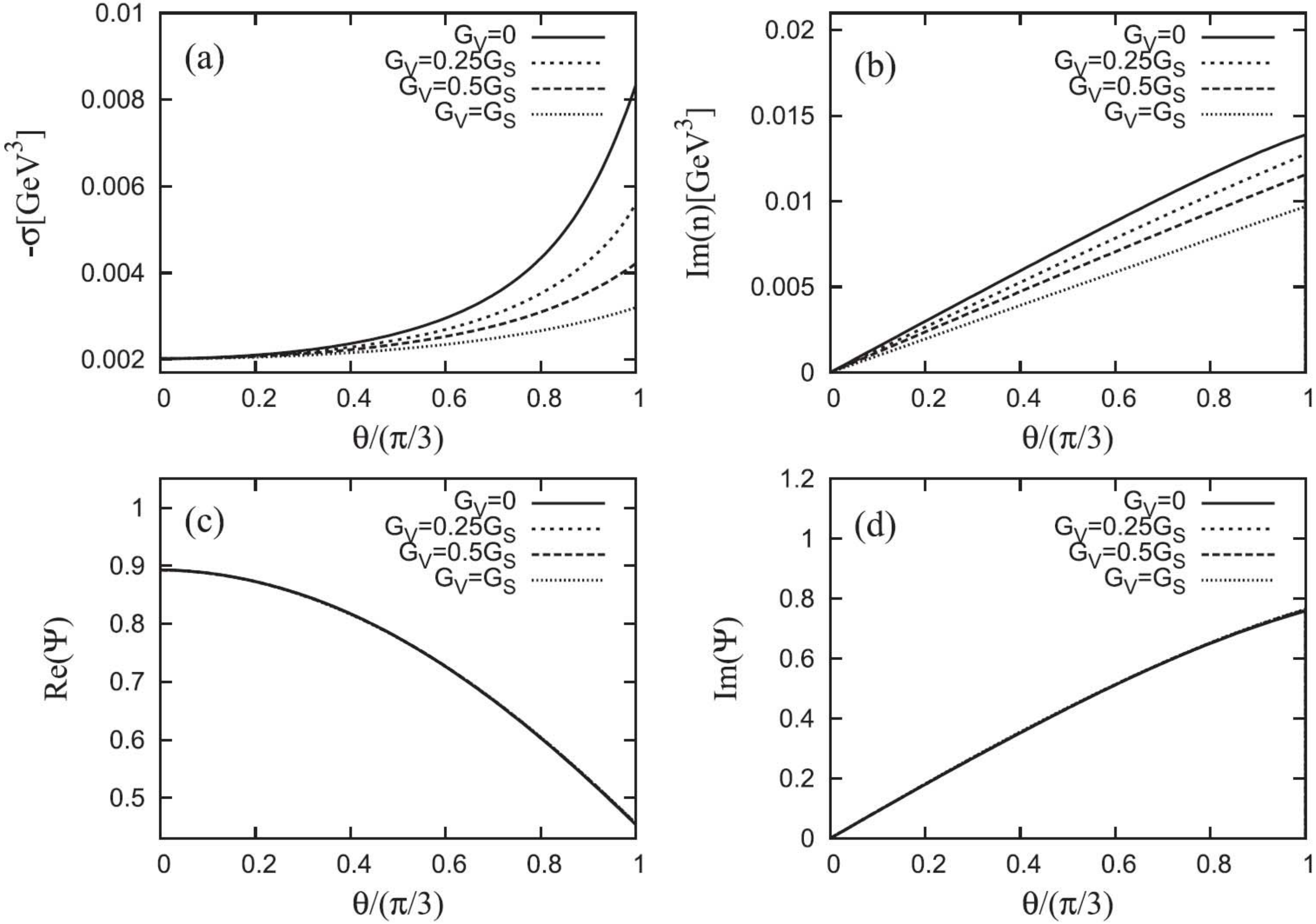
We can determine the strength of **vector type interaction.**



Our result 2-2



We can determine the strength of **vector type interaction**.



Y. Sakai, K. K, H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Rev. D **78** (2008) 076071.

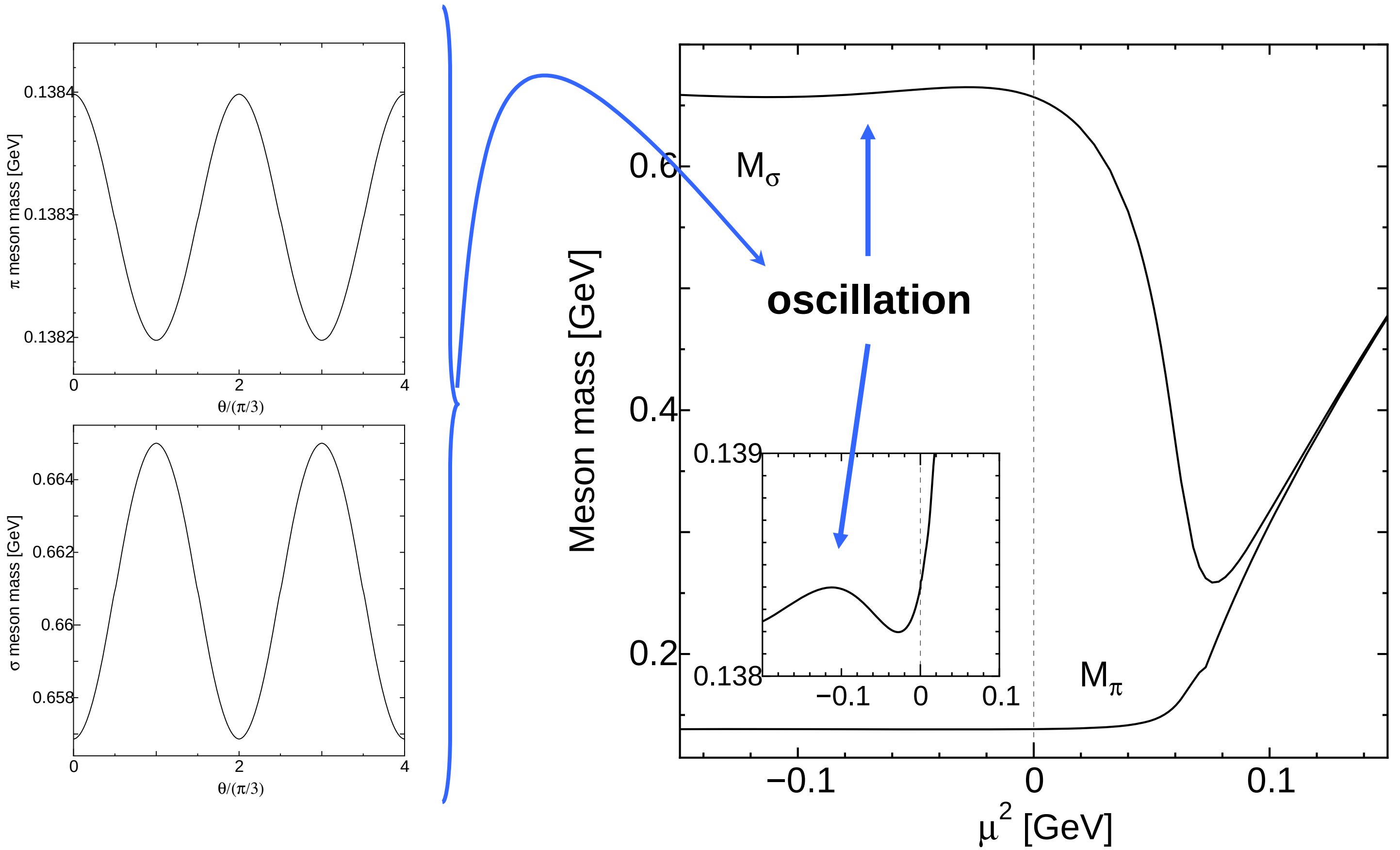
Our result 3-1

K. K, M. Matsuzaki, H. Kouno, Y. Sakai and M. Yahiro, in preparation.

Meson has **RW periodicity** as same as chiral condensate.

T=160 MeV

$$\theta = \mu_I / T$$

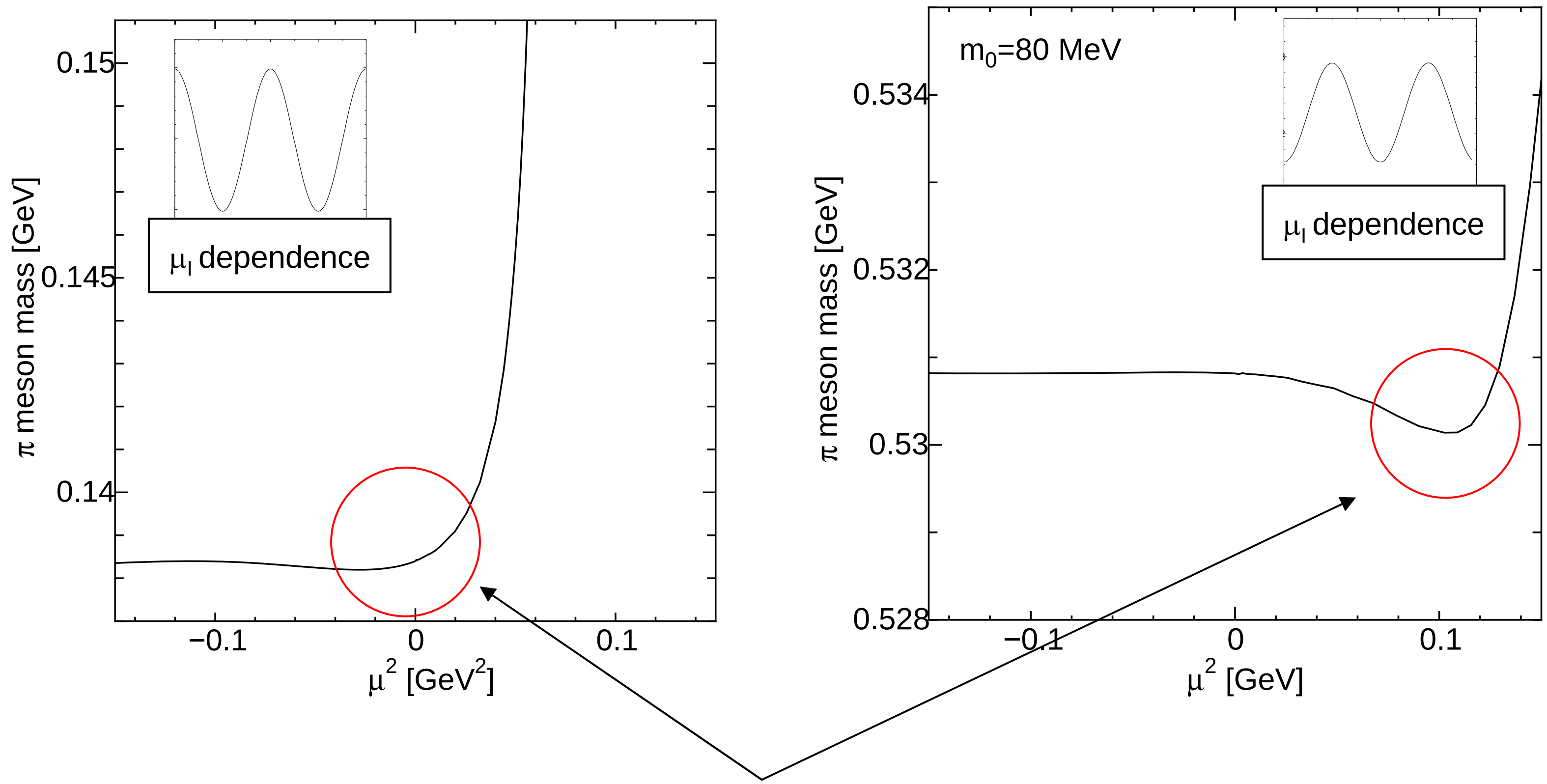


Our result 3-2

K. K, M. Matsuzaki, H. Kouno, Y. Sakai and M. Yahiro, in preparation.

Meson mass is **good indicator** of the consistency about current mass.

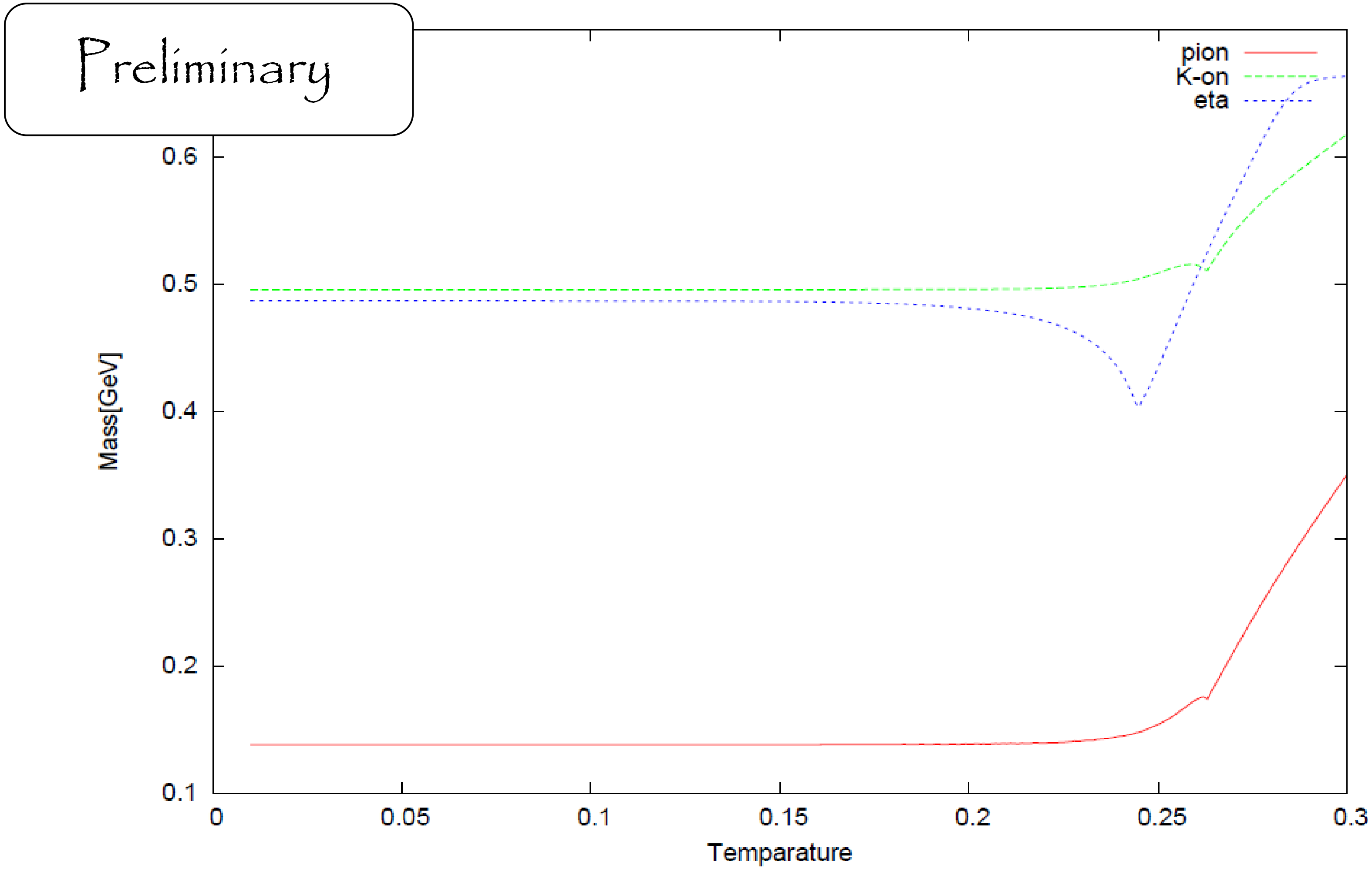
T=160 MeV



The Qualitatively difference appears.

Our result 3+

T. Matsumoto, K.K., H. Kouno, M. Matsuzaki and M. Yahiro, in preparation.



The result only in real μ

P. Costa et al. arXiv:hep-ph/0807.2134.

Three flavor system can be investigated by same method.

Our result 3-3

In resent our results, we really fit parameters by comparing with lattice data,
chiral condensate, number density, values of modified Polyakov-loop and transition lines,

But, we did not check the validity in the moment.

(We neglect some terms and thermodynamical effects)

↑

The **meson mass** is also the **indicator** of reliability of the parameter set.

Because the meson mass is sensitive for parameter set.

(Therefore parameters are fitted by reproducing the empirical values at $T=\mu=0$ in the usual NJL type model)

Therefore, we must check the validity by using meson masses in future.

(But more detail investigation of the parameter set is necessary)

Our result 4-1

Y. Sakai, K. K, H. Kouno, M. Matsuzaki and M. Yahiro, in preparation.

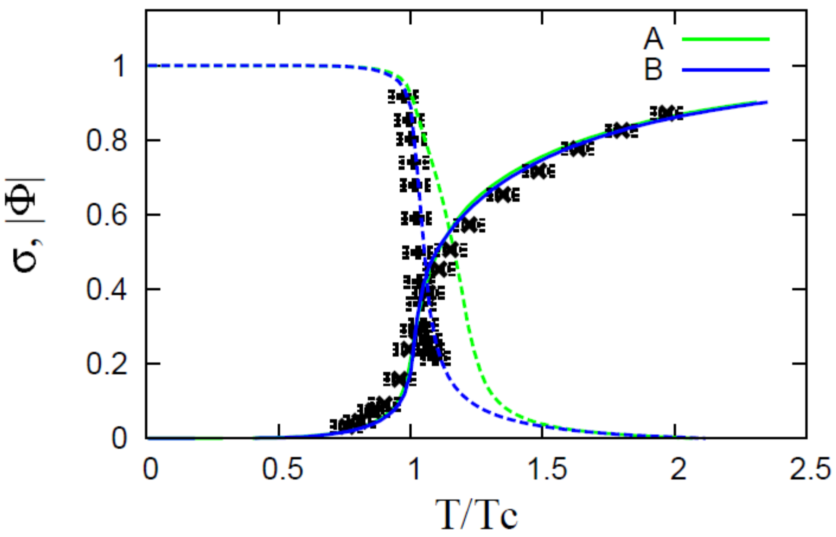
Parameter set obtained by our approach and results.

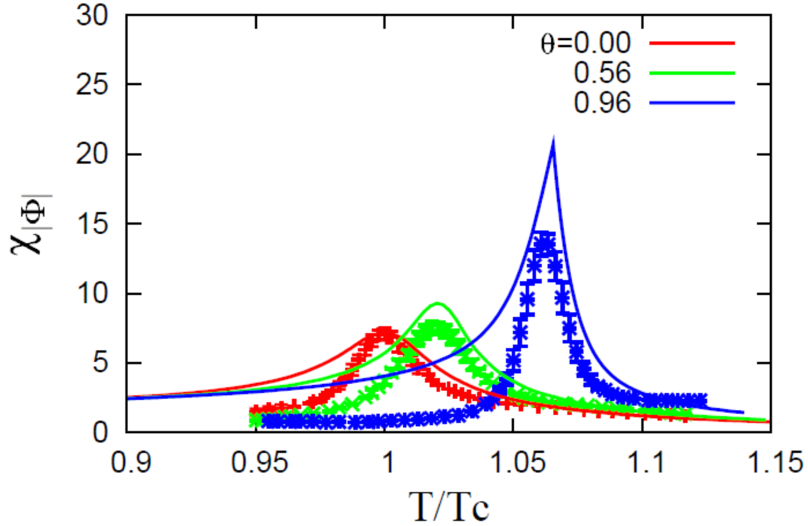
set	G_s	G_{s8}	G_v
A	5.498GeV^{-2}	0	0
B	4.761GeV^{-2}	403.89GeV^{-8}	0
C	4.761GeV^{-2}	403.89GeV^{-8}	4.761GeV^{-2}

TABLE II: Summary of the parameter sets in the PNJL calculations. The parameters Λ, m_0 and T_0 are common among the three sets; $\Lambda = 631.5$ MeV, $m_0 = 5.5$ MeV and $T_0 = 212$ MeV.

Lattice data:

F. Karsch, Lect. notes Phys. **583** (2002) 209.
M. Kaczmarek and F. Zantow, Phys. Rev. D **71**, (2005) 114510.
L. K. Wu, X. Q. Luo and H. S. Chen, Phys. Rev. D **76** (2007) 034505.





Good agreement

$$(\mu_R = \mu_I = 0)$$

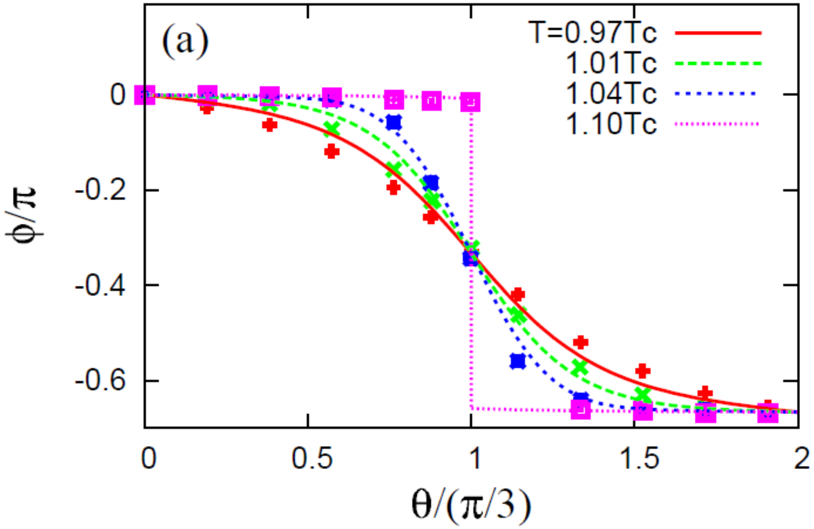
Our result 4-2

Y. Sakai, K. K, H. Kouno, M. Matsuzaki and M. Yahiro, in preparation.

Parameter set obtained by our approach and results.

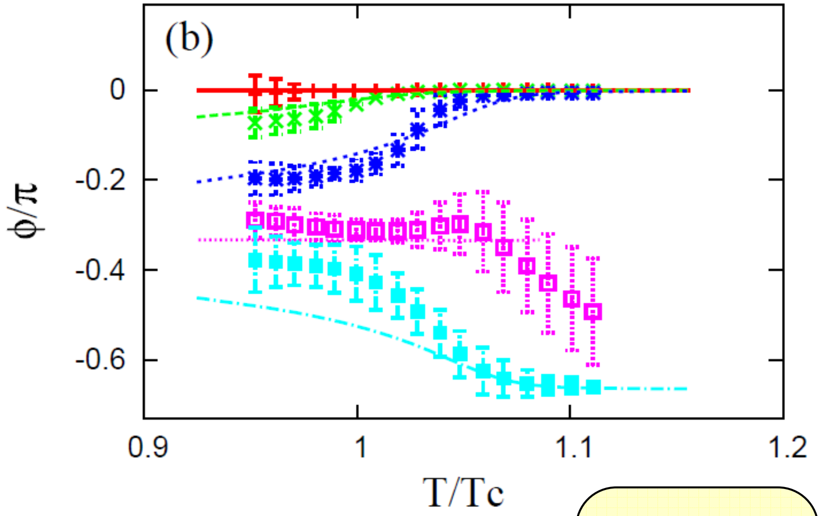
set	G_s	G_{s8}	G_v
A	$5.498\mathrm{GeV}^{-2}$	0	0
B	$4.761\mathrm{GeV}^{-2}$	$403.89\mathrm{GeV}^{-8}$	0
C	$4.761\mathrm{GeV}^{-2}$	$403.89\mathrm{GeV}^{-8}$	$4.761\mathrm{GeV}^{-2}$

TABLE II: Summary of the parameter sets in the PNJL calculations. The parameters Λ, m_0 and T_0 are common among the three sets; $\Lambda = 631.5$ MeV, $m_0 = 5.5$ MeV and $T_0 = 212$ MeV.



Lattice data:

P. de Forcrand and O. Philipsen, Nucl. Phys. **B642** (2002) 290.
L. K. Wu, X. Q. Luo and H. S. Chen, Phys. Rev. D **76** (2007) 034505.



Good agreement at several θ.

$\theta = 0$

$\theta = 0.4$

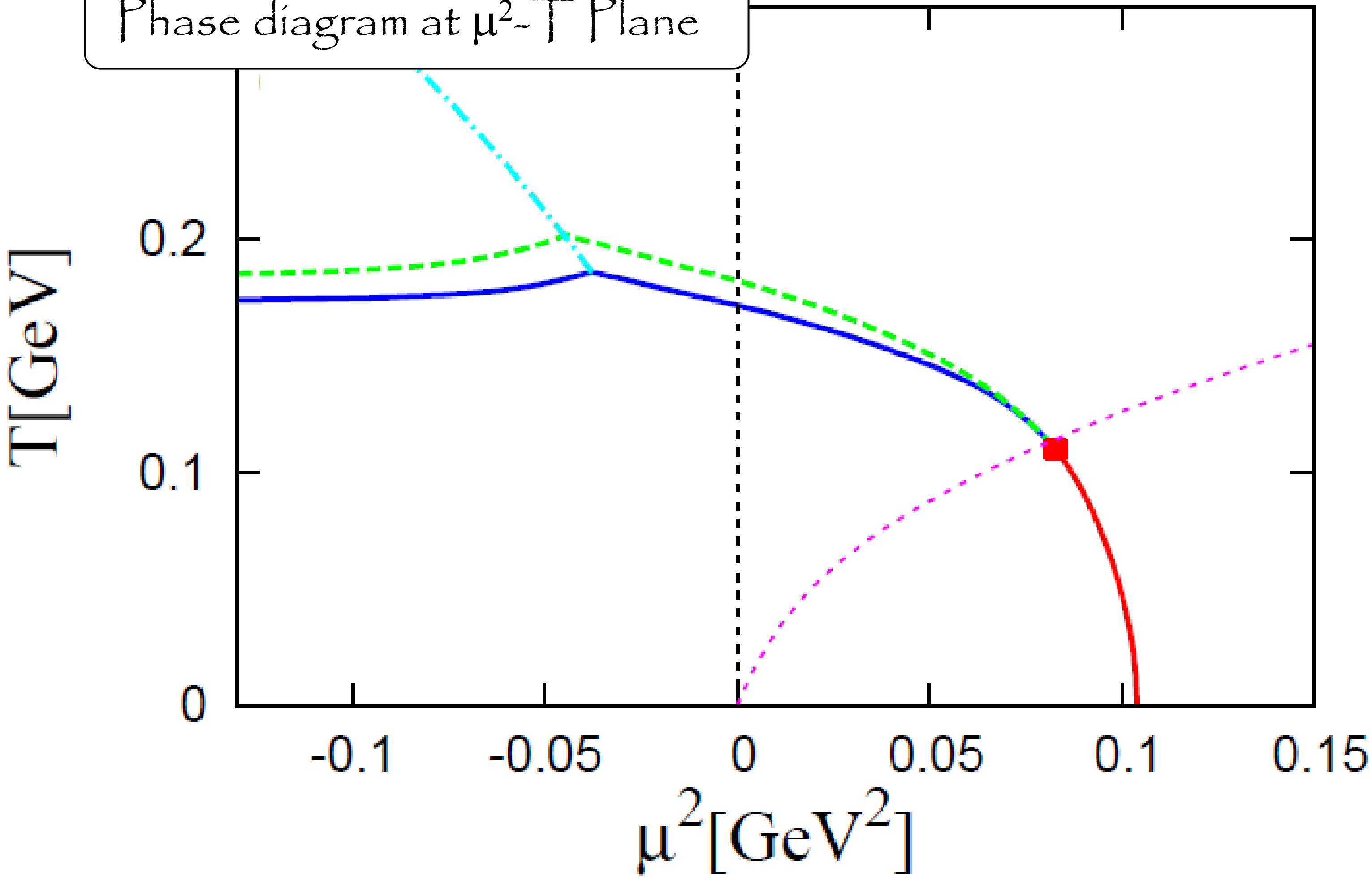
$\theta = 0.8$

$\theta = 1.0$

Our result 4-3

Y. Sakai, K. K, H. Kouno, M. Matsuzaki and M. Yahiro, in preparation.

Phase diagram at μ^2-T Plane



Summary and future work

Summary

- **The PNJL model can reproduce** the **RW periodicity**.

- The **strength of vector type interaction can be determined**.

- **Meson masses have RW periodicity** like as other thermodynamical value.

  (Chiral condensate, Pressure and so on)

- **Small current mass** is **necessary**

  because the behavior of oscillation of meson masses at imaginary μ

  is **directly affect** the behavior of meson mass at real μ.

Quantitatively discussions of

the QCD phase diagram is enabled by using

the **imaginary chemical potential matching approach**.

But more detail calculation is necessary.

END