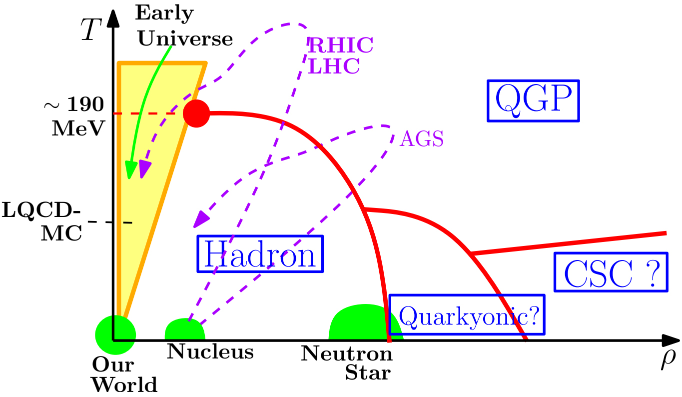
Status of Strong Coupling Lattice QCD in Exploring QCD Phase Diagram

KOHTARHO. MIURA

YITP

12/26(Fri), 2008, Talk in Kyusyu Univ.

QCD Phase Diagram




Lattice MC.
Models
SC-LQCD

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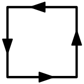
Lattice QCD Action (Pure Glue)

- **Plaquette**



$$U_{\nu\rho,x} \sim e^{ia^2 g\ G_{\nu\rho,x}}$$

- **Action for Pure Glue**

$$S_G = \sum_{\nu\rho,x} \frac{2N_c}{g^2} \left[1 - \frac{\mathrm{tr}_c}{2N_c}\left[U_{\nu\rho,x} + U^\dagger_{\nu\rho,x}\right]\right] \to \frac{1}{4}\int d^4x\ G_{\nu\rho,x} G^{\nu\rho}_x \qquad (1)$$

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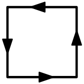

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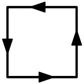

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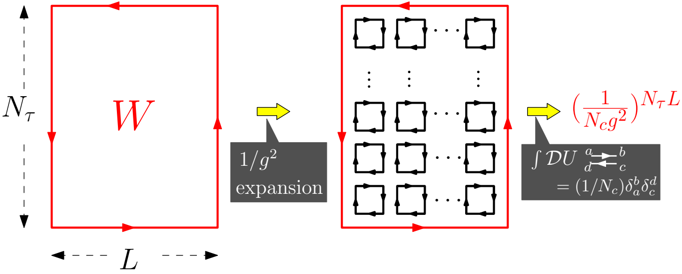

$$\langle W[U] \rangle \propto \int \mathcal{D}U \; W[U] \; \exp\left[-S_G[U_\Box]\right] \simeq \exp[-N_\tau \mathcal{V}]$$

$$\mathcal{V} = L \log[N_c g^2] \qquad (2)$$

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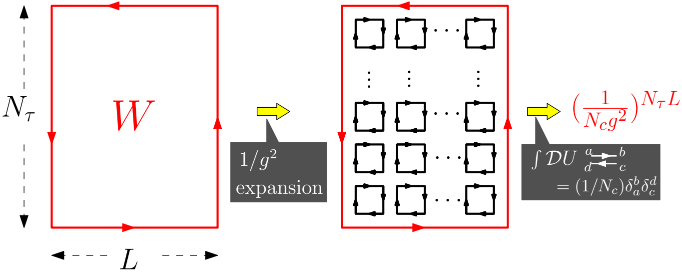

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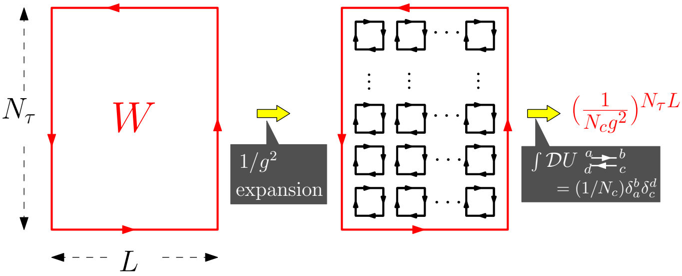

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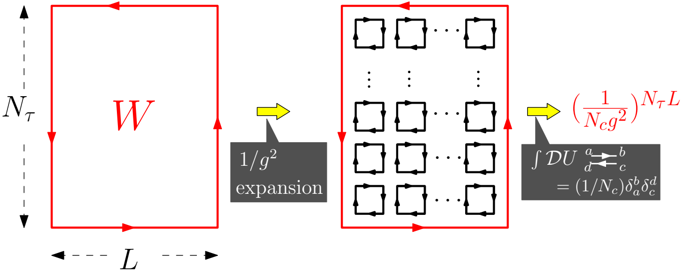

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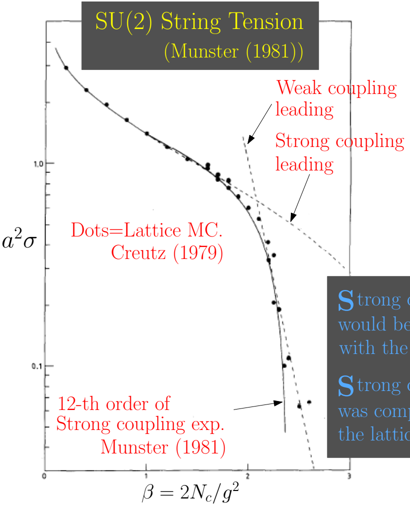
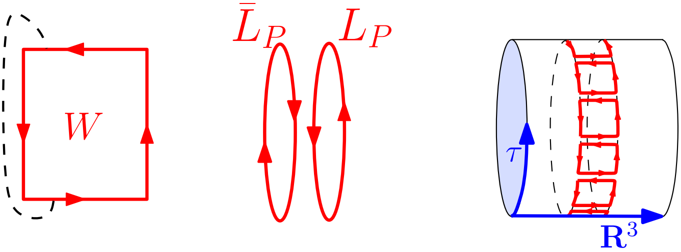
String tension



Strong coupling regime would be smoothly connected with the perturbative regime.

Strong coupling expansion was complimentary to the lattice MC, and vice versa.

Polyakov Loop

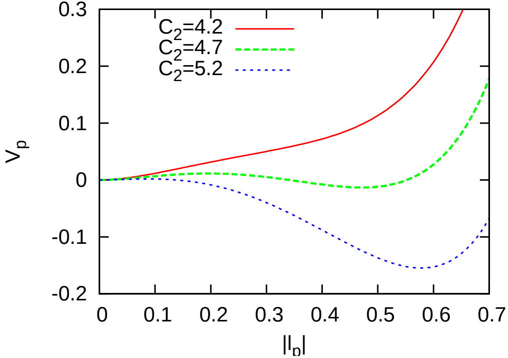


Potential of Polyakov Loop (SU($N_c = 3$))

SU(2): Polonyi,Szlachanyi(1982) , SU(3): Gross, J. Bartholomew, and D. Hochberg(1983)

c.f. PNJL (Fukushima (2003))

$$\mathcal{V}_P/T = -2d\ e^{-a\sigma/T} N_c^2 \bar{l}_P l_P - \log\left[1 - 6\bar{l}_P l_P - 3(\bar{l}_P l_P)^2 + 4(l_P^3 + \bar{l}_P^3)\right] \quad (3)$$



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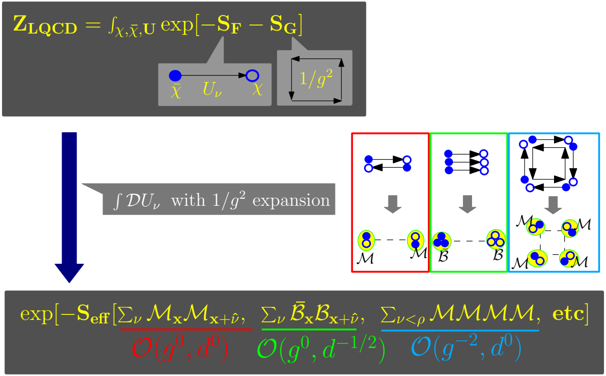
$1/g^2$ & $1/d$ expansion

Pioneering Works: Kawamoto,Smit ('81), Kluberg-Stern,Moreo,Napoly,Peterson('81)

1/d expansion: Kluberg-Stern,Moreo,Peterson('83)



Hadron Mass Spectrum

Staggered Hadrons: Kluberg-Stern,Moreo,Peterson('83), Golterman,Smit('85)

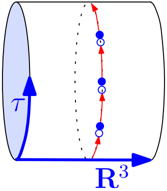
	Strong coupling expansion		Monte Carlo [8]	Monte Carlo [9]	Physical values
g^2N	∞ [1]	3	3	3.15	
m_π	input		input	input	$m_\pi = 140$
m_ρ	input (780)		input (750)	730 ± 90	$m_\rho = 780$
$M_2(A_1)$	1010	930	1120	1190 ± 90	$m_{A_1} = 1100$
$M_3(S)$	1160	930	970	660 ± 50	$m_\delta = 980$
m_B	1300	1040	{1000, 1700}	920 ± 100	$m_N = 940$, $m_\Delta = 1240$
f_π	190	190	177	134	95
m_q	8	7	8	7	
a^{-1}	440	524	1500	input (730)	

Kluberg-Stern-Morel-Petersson(1983)

Strong Coupling Expansion is consistent with the lattice MC and experiments !!

T and μ

- **Polyakov Gauge**



$$U_{0,\mathbf{x}} = \mathrm{diag}\{e^{i\theta^1_{\mathbf{x}}T}, e^{i\theta^2_{\mathbf{x}}T}, e^{i\theta^3_{\mathbf{x}}T}\}$$

- **Lattice Chemical Potential** Karsch,Hasenfratz ('83)

$$U_0 \to e^{\mu} U_0 \ , \qquad (c.f.\ iA_0 \to iA_0 + \mu) \tag{4}$$

T and μ

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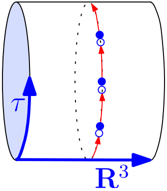

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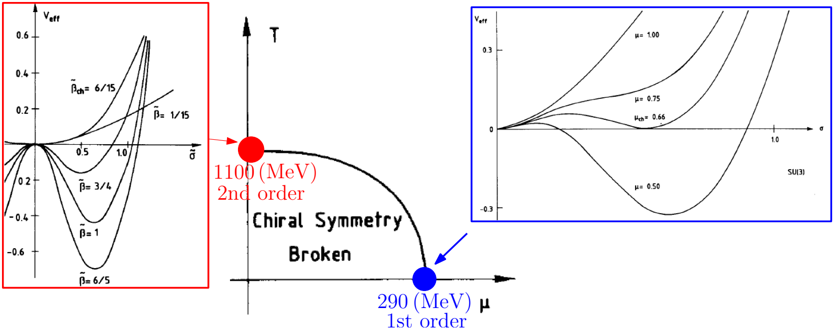
Phase Diagram

Damgaard-Kawamoto-Shigemoto (1986)

Damgaard-Hochberg-Kawamoto (1985)



Some Comments

- **Effective Potential**
  Damgaard,Kawamoto,Shigemoto('86), Faldt,Petersson ('86)

$$V_{\rm eff} = \frac{d}{4N_c}\sigma^2 - T\log\left[\frac{\sinh[(N_c+1)E/T]}{\sinh[E/T]} + 2\cosh[N_c\mu/T]\right] \tag{5}$$

- **Phase Diagrams**
  - Bilic,Karsch,Redlich (1992)
  - Bilic,Demeterfi,Peterson (1992)
  - Bilic,Cleymens (1995)
- **Related Models**
  - Ilgenfritz,Kripfganz(1985)
  - Gocsh,Ogilve (1986)
- **Monomer-Dimer-Polymer**
  - Dagotto,Moreo,Wolf (1986,87)
  - Karsch,Mutter (1990)

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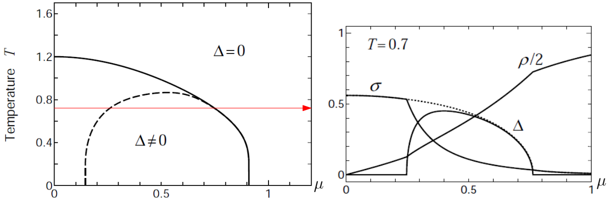
Strong Coupling Limit SU(2)



Nishida-Fukushima-Hatsuda (2003) SU($N_c = 2$), Strong coupling limit

- Pauli-Gursey ($\sigma \leftrightarrow \Delta$) symmetry at $(m_0, \mu) = (0, 0)$. m_0 favors σ, μ favors Δ. Saturation effect.
- Similar phase diagram is obtained in SU($N_c = 3$) with isospin chemical pot. ($\sigma \leftrightarrow \pi$) (Nishida '04).
- The diquark has not been realized in SU($N_c = 3$). (*c.f.* Diquarks in SU(3): Azcoiti et al. (2003))

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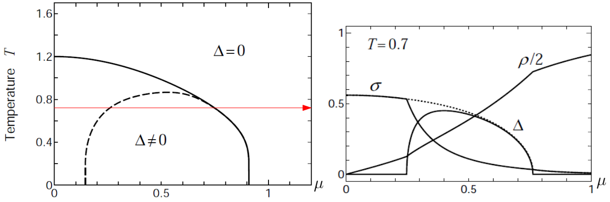

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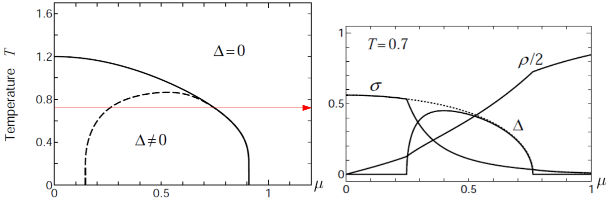

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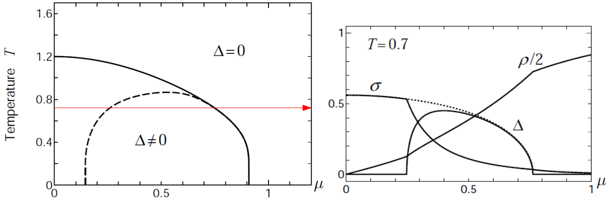

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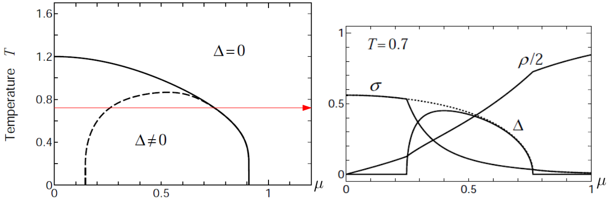

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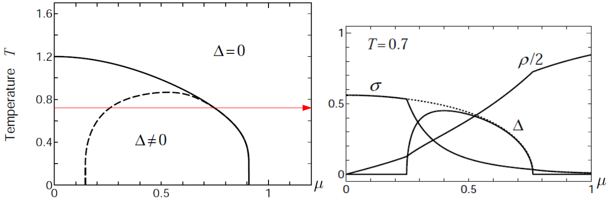

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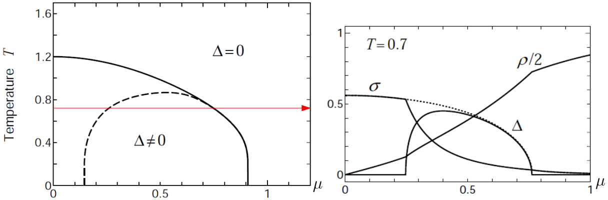

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Strong Coupling Limit SU(3)

Fukushima('04), Nishida('04), Kawamoto-Miura-Ohnishi-Ohnuma('05)

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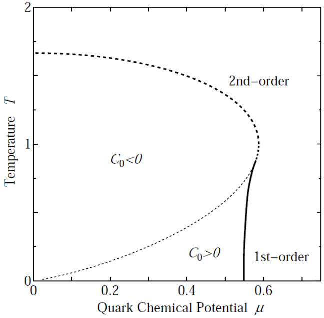

Kawamoto-Miura-Ohnishi-Ohnuma ('07)
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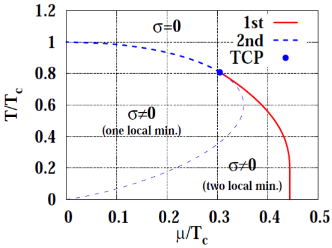

1st and 2nd transitions with tri-critical point appear!!

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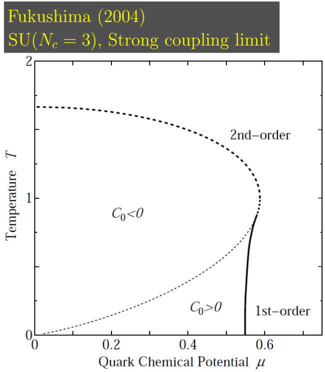

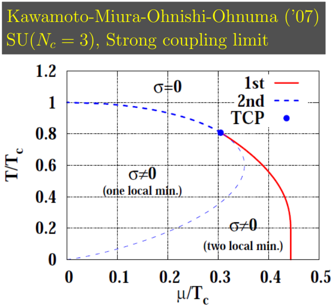

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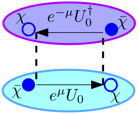
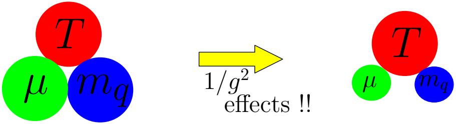
Energy scale modifications due to $1/g^2$

- **A Plaquette Effect and MFA**

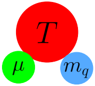
A Diagram with $1/g^2$	Mean Fields	Physical Meaning
	$\varphi_\tau \sim \langle e^{\mu}\bar{\chi}U_0\chi - e^{-\mu}(h.c.)\rangle$	Quark mass (m_q) suppression
	$\phi_\tau \sim \langle e^{\mu}\bar{\chi}U_0\chi + e^{-\mu}(h.c.)\rangle$	Quark density (μ suppression)

- **Energy scale**

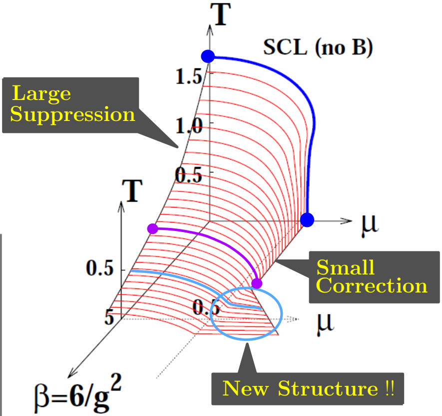


Phase Diagram Evolution with $\beta = 2N_c/g^2$

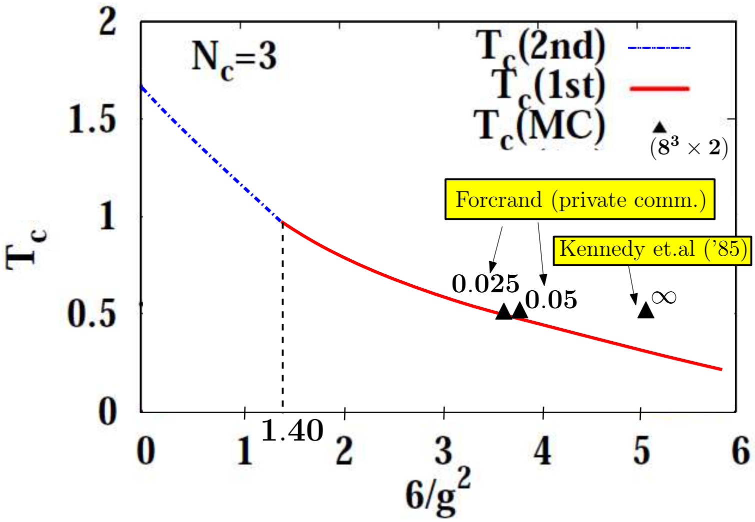
Miura-Kawamoto-Ohnishi, Preliminary



$\mu^{\rm cri}_{T\ 0}/T^{\rm cri}_{\mu=0}$	
> 2.0	(real world)
0.33	Fukushima ('04) Nishida ('04)
~ 1.0	Present ($\beta \sim 3.2$)
≥ 1.0	MC (Forcrand-Philipsen, Fodor-Katz, ...)



Consistency check



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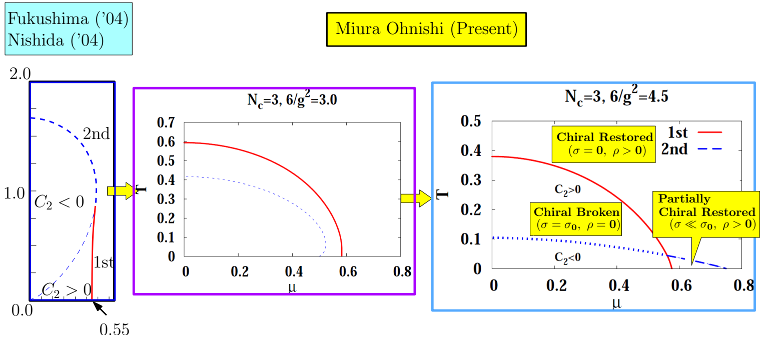
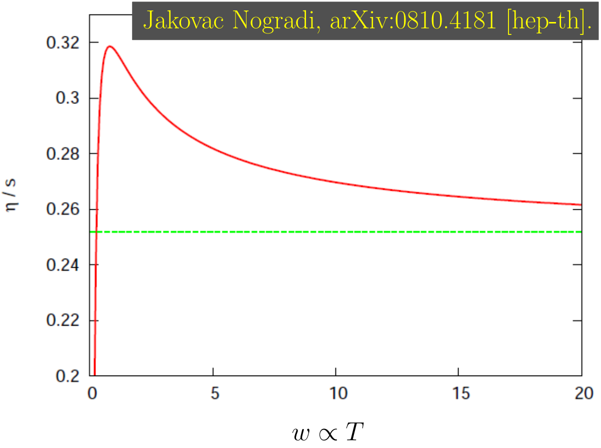


$$\beta \geq \beta_c = \frac{2N_c^2}{d}\mu_c^{(1\mathrm{st})} \qquad (6)$$

Shear Viscosity based on SCExp. in Pure Glue



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Summary

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- Idea source for the model buildings.

Recent developments

- Precise structure of the phase diagram at the strong coupling limit (SU(2) and SU(3)).
- The phase diagram evolution with the finite coupling.
- Shear viscosity in pure glue $\eta/s \sim 0.25$.
- Meson mass scalings due to T and μ effects (Miura,Kawamoto Ohnishi ('08)), Banks-Casher relations in SU(2) SC-LQCD at $g \to \infty$ (Fukushima ('08)),Diquark (Azcoiti et al. ('03))etc.

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Future Developments

Key Words

- Imaginary chemical potential.
- Comparison with the lattice MC in the scaling region at $\mu = 0$.
- Introducing the SC-LQCD inspired interactions to models.
- More sophisticated formulations for diquarks and Viscosity.
- Finite T glueball.
- Phase diagram for the deconfinement transition.
- The density creation in the chiral broken phase.