Status of Strong Coupling Lattice QCD in Exploring QCD Phase Diagram

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YITP

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KOHTARHO. MIURA

Status of Strong Coupling Lattice QCD in Exploring QCD Phase Diagram
Introduction (5 min.)
Confinement and Deconfinement (10 min.)
Chiral Phase Transition (10 min.)
Recent Developments (20 min)
Summary and Future Developments

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Lattice MC.  SC-LQCD  Models
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   - Lattice QCD Action (Pure Glue)
   - Wilson Loop
   - Polyakov Loop
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   - $1/g^2$ & $1/d$ expansion
   - Hadron Mass Spectrum
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Lattice QCD Action (Pure Glue)

- **Plaquette**

\[ U_{\nu\rho,x} \sim e^{ia^2gG_{\nu\rho,x}} \]

- **Action for Pure Glue**

\[
S_G = \sum_{\nu\rho,x} \frac{2N_c}{g^2} \left[ 1 - \frac{\text{tr}_c}{2N_c} \left[ U_{\nu\rho,x} + U_{\nu\rho,x}^\dagger \right] \right] \to \frac{1}{4} \int d^4x \ G_{\nu\rho,x} G_{x}^{\nu\rho}
\]  \hspace{1cm} (1)
Lattice QCD Action (Pure Glue)

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  \]
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**Lattice QCD Action (Pure Glue)**

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  \]

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Status of Strong Coupling Lattice QCD in Exploring QCD Phase Diagram
Wilson Loop (Wilson (1974))

\[ \langle W[U] \rangle \propto \int D U \ W[U] \ \exp[-S_{G[U]}] \simeq \exp[-N_{\tau} \mathcal{V}] \]

\[ \mathcal{V} = L \log[N_c g^2] \tag{2} \]
Wilson Loop (Wilson (1974))

\[ \langle W[U] \rangle \propto \int DU \ W[U] \ \exp[-S_G[U_{\square}]] \simeq \exp[-N_\tau \mathcal{V}] \]

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Lattice QCD Action (Pure Glue)
Wilson Loop
Polyakov Loop

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\]

\[
\mathcal{V} = L \log[N_c g^2]
\]

\[
\int DU \frac{a}{d} = \frac{1}{N_c} \delta^b_a \delta^d_c
\]
String tension

**SU(2) String Tension (Munster (1981))**

- **Weak coupling leading**
- **Strong coupling leading**

Dots = Lattice MC.

Creutz (1979)

\[
\beta = 2N_c/g^2
\]

12-th order of Strong coupling exp. Munster (1981)

Strong coupling regime would be smoothly connected with the perturbative regime.

Strong coupling expansion was complimentary to the lattice MC, and vice versa.
Potential of Polyakov Loop ($SU(N_c = 3)$)


c.f. PNJL (Fukushima (2003))

$$V_P / T = -2d \ e^{-a\sigma / T} N_c^2 \bar{l}_P l_P - \log \left[ 1 - 6\bar{l}_P l_P - 3(\bar{l}_P l_P)^2 + 4(l_P^3 + \bar{l}_P^3) \right]$$ (3)
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**Pioneering Works:** Kawamoto, Smit (’81), Kluberg-Stern, Moreo, Napoly, Peterson (’81)

**1/d expansion:** Kluberg-Stern, Moreo, Peterson (’83)

\[
Z_{\text{LQCD}} = \int x, \bar{x}, U \exp[-S_F - S_G]
\]

\[
\exp[-S_{\text{eff}}[\sum_\nu M_x M_{x+\hat{p}}, \sum_\nu \bar{B}_x B_{x+\hat{p}}, \sum_{\nu<\rho} M.M.M.M.M., \text{etc}]]
\]

\[
\mathcal{O}(g^0, d^0) \quad \mathcal{O}(g^0, d^{-1/2}) \quad \mathcal{O}(g^{-2}, d^0)
\]
**Staggered Hadrons**: Kluberg-Stern, Moreo, Peterson (’83), Golterman, Smit (’85)

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<td>970</td>
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<td>970</td>
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<td>$m_B$</td>
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<td>1120</td>
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<td>1500</td>
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Kluberg-Stern-Morel-Petersson (1983)

Strong Coupling Expansion is consistent with the lattice MC and experiments!!
Polyakov Gauge

\[ U_{0,x} = \text{diag}\{e^{i\theta_1 x T}, e^{i\theta_2 x T}, e^{i\theta_3 x T}\} \]

Lattice Chemical Potential Karsch, Hasenfratz ('83)

\[ U_0 \rightarrow e^{\mu} U_0 \quad (\text{c.f. } iA_0 \rightarrow iA_0 + \mu) \]
**T and μ**

- **Polyakov Gauge**

  \[ U_{0,x} = \text{diag}\{e^{i\theta^1_x T}, e^{i\theta^2_x T}, e^{i\theta^3_x T}\} \]

- **Lattice Chemical Potential** Karsch, Hasenfratz ('83)

  \[ U_0 \rightarrow e^{\mu} U_0 , \quad (c.f. \ iA_0 \rightarrow iA_0 + \mu) \quad (4) \]
Phase Diagram

**Damgaard-Kawamoto-Shigemoto (1986)**

- **V_{eff}**
- \( \tilde{\beta} = 6/15 \)
- \( \tilde{\beta} = 1/15 \)
- \( \tilde{\beta} = 3/4 \)
- \( \tilde{\beta} = 1 \)
- \( \tilde{\beta} = 6/5 \)

**Damgaard-Hochberg-Kawamoto (1985)**

- **\( V_{eff} \)**
- \( \mu = 1.00 \)
- \( \mu = 0.75 \)
- \( \mu = 0.65 \)
- \( \mu = 0.50 \)

1100 (MeV) 2nd order

290 (MeV) 1st order
Some Comments

- **Effective Potential**
  Damgaard, Kawamoto, Shigemoto ('86), Faldt, Petersson ('86)

  \[
  V_{\text{eff}} = \frac{d}{4N_c} \sigma^2 - T \log \left[ \frac{\sinh[(N_c + 1)E/T]}{\sinh[E/T]} + 2 \cosh[N_c\mu/T] \right]
  \]  

  (5)

- **Phase Diagrams**
  - Bilic, Karsch, Redlich (1992)
  - Bilic, Demeterfi, Peterson (1992)
  - Bilic, Cleymens (1995)

- **Related Models**
  - Ilgenfritz, Kripfganz (1985)
  - Gocsh, Ogilve (1986)

- **Monomer-Dimer-Polymer**
  - Dagotto, Moreo, Wolf (1986, 87)
  - Karsch, Mutter (1990)
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5 Summary and Future Developments
Pauli-Gursey ($\sigma \leftrightarrow \Delta$) symmetry at $(m_0, \mu) = (0, 0)$. $m_0$ favors $\sigma$, $\mu$ favors $\Delta$. Saturation effect.

Similar phase diagram is obtained in SU($N_c = 3$) with isospin chemical pot. ($\sigma \leftrightarrow \pi$) (Nishida ’04).

The diquark has not been realized in SU($N_c = 3$). (c.f. Diquarks in SU(3): Azcoiti et al. (2003))
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Strong Coupling Limit SU(3)

Fukushima (’04), Nishida (’04), Kawamoto-Miura-Ohnishi-Ohnuma (’05)

1st and 2nd transitions with tri-critical point appear!!
Fukushima ('04), Nishida ('04), Kawamoto-Miura-Ohnishi-Ohnuma ('05)

1st and 2nd transitions with tri-critical point appear!!
Energy scale modifications due to $1/g^2$

- **A Plaquette Effect and MFA**

  A Diagram with $1/g^2$

  \[
  \chi e^{-\mu U_0^\dagger} \chi \\
  \chi e^{\mu U_0} \chi
  \]

  Mean Fields

  \[
  \varphi_\tau \sim \langle e^{\mu} \bar{\chi} U_0 \chi - e^{-\mu} (h.c.) \rangle \\
  \phi_\tau \sim \langle e^{\mu} \bar{\chi} U_0 \chi + e^{-\mu} (h.c.) \rangle
  \]

  Physical Meaning

  Quark mass ($m_q$) suppression

  Quark density ($\mu$ suppression)

- **Energy scale**

  \[
  T \quad \mu \quad m_q
  \]

  1/$g^2$ effects !!
Phase Diagram Evolution with $\beta = 2N_c/g^2$

Miura-Kawamoto-Ohnishi, Preliminary

- $\mu T_{0}/T_{\mu=0}^{\text{cri}}$
  - $> 2.0$ (real world)
  - $0.33$ Fukushima ('04)
  - $Nishida ('04)$
  - $\sim 1.0$ Present ($\beta \sim 3.2$)
  - $\geq 1.0$ MC (Forcrand-Philipsen, Fodor-Katz, ...)

- Large Suppression
- Small Correction
- New Structure!!
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Consistency check

\[
T_c(1st) \quad T_c(2nd) \quad T_c(MC)
\]

\(N_c = 3\)

Forcrand (private comm.)
Kennedy et.al (’85)

\((8^3 \times 2)\)

0.025 0.05 ∞
Phase Diagram Evolution with $\beta = 2N_c/g^2$

\[ \beta \geq \beta_c = \frac{2N_c^2}{d} \mu_c^{(1st)} \]  

\( C_2 < 0 \)  
\( C_2 \geq 0 \)  
\( C_2 > 0 \)

Fukushima ('04)  
Nishida ('04)

Miura Ohnishi (Present)
Shear Viscosity based on SCExp. in Pure Glue

\[ \eta \propto T \]

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Status

- An instructive guide for the lattice MC. In particular, “Beyond Sign Problem” may be urgently required.
- Idea source for the model buildings.

Recent developments

- Precise structure of the phase diagram at the strong coupling limit (SU(2) and SU(3)).
- The phase diagram evolution with the finite coupling.
- Shear viscosity in pure glue $\eta/s \sim 0.25$.
- Meson mass scalings due to $T$ and $\mu$ effects (Miura, Kawamoto Ohnishi ('08)), Banks-Casher relations in SU(2) SC-LQCD at $g \to \infty$ (Fukushima ('08)), Diquark (Azcoiti et al. ('03)) etc.
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Future Developments

Key Words

- Imaginary chemical potential.
- Comparison with the lattice MC in the scaling region at $\mu = 0$.
- Introducing the SC-LQCD inspired interactions to models.
- More sophisticated formulations for diquarks and Viscosity.
- Finite $T$ glueball.
- Phase diagram for the deconfinement transition.
- The density creation in the chiral broken phase.