Baryons and baryonic matter in holographic QCD


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INTRODUCTION


“unified meson theory”

- Meson spectra of (axial) vector mesons.  
- Hidden local symmetry.  [Bando, et al., 1985]  
- Vector meson dominance.  [Sakurai, 1969]  
- KSRF relation.  [Kawarabayashi, Suzuki, Riazuddin, and Fayyazuddin, 1966]  

- Classical supergravity is dual with large-Nc QCD.
- As a general property of large-Nc QCD, a baryon does not directly appear as a dynamical degrees of freedom.  

How to describe baryons in the large- $N_c$ holographic model?

* Chiral soliton picture : baryon as a soliton of meson field theory induced by holographic QCD ("brane-induced Skyrmion").

* Baryonic matter as a brane-induced Skyrmion on $S^3$. 
Holographic QCD
(Sakai-Sugimoto model)

Baryons in holographic QCD

Baryonic matter in holographic QCD
Holographic QCD

Construction of massless QCD (quarks and gluons) on D4/D8/D8-branes.

\[ [T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)] \]

- Index on D4 brane (color)
- Index on D8 brane (flavor)

\[ \psi_{L}^{a,i}(x_{0-3}) \]

: quark (L)

\[ A_{\mu=0-3}^{a,b} \]

: gluon

\[ \psi_{R}^{a,i}(x_{0-3}) \]

: quark (R)

- 4-4
- 4-8

10dim.
Probe approximation with $N_C >> N_f$

Mass of $Dp$-brane is proportional to its sheets number.

$D4 \times N_c \longrightarrow$ SUGRA background
$D8, \overline{D8} \times N_f \longrightarrow$ "probe"

$D4$-brane $\times N_c$

$D8 \times N_r$
$\overline{D8} \times N_r$

mode expansion

$pion$

(axial) vector mesons $\rho, \rho', \rho'', \ldots$

Baryon does not directly appear as a general property of large-$N_c$ QCD.

Baryons in holographic QCD

* chiral perturbation theory (chiral symmetry × Lorentz invariance)

2-derivative term
\[ \mathcal{L}_2 : \text{tr}(L_\mu L_\mu) \]

4-derivative term
\[ \mathcal{L}_4 : \text{tr}[L_\mu, L_\nu]^2, \text{tr} \{L_\mu \times L_\nu\}^2, \text{tr}(\partial_\mu L_\mu)^2 \]

Effective action of D8 brane
\[ \kappa \int d^4x dz \text{tr} \left( \frac{1}{2} K^{-1/3} F_{\mu\nu} F_{\mu\nu} + K F_{\mu \tau} F_{\mu \tau} \right) \]

Instability of Skyrme soliton.

Topological picture for baryon is also supported by superstring theory.

Baryon as a chiral soliton

\[ \text{c.f.} \quad \text{Baryon as an instanton in 5dim. gauge theory on D8.} \]
[D.K. Hong, M. Rho, H.-U. Yee, and P. Yi, PRD76, 061901(2007)]
4dim. meson effective action from holographic QCD (pions and \( \rho \)-mesons)

\[
\begin{align*}
S_{D8}^{DBI} - S_{D8}^{DBI}|_{A_M \to 0} &= \kappa \int d^4x dz tr \left\{ \frac{1}{2} K(z)^{-\frac{1}{2}} F_{\mu\nu} F_{\mu\nu} + K(z) F_{\mu z} F_{\mu z} \right\} \\
&= \frac{f_\pi^2}{4} \int d^4x tr (L_\mu L_\mu) \quad \text{(chiral term)} \\
&+ \frac{1}{2} \int d^4x tr \left[ L_\mu L_\nu \right]^2 \quad \text{(Skyrme term)} \\
&+ \frac{1}{2} \int d^4x \left( \partial_\mu \rho_\nu - \partial_\nu \rho_\mu \right)^2 \quad \text{(\( \rho \)-kinetic term)} \\
&+ m_\rho^2 \int d^4x \left( \rho_\mu \rho_\mu \right) \quad \text{(\( \rho \)-mass term)} \\
&+ \frac{1}{2} g_3 \rho \left[ -i \right] \int d^4x tr \left\{ \left( \partial_\mu \rho_\nu - \partial_\nu \rho_\mu \right) \left[ \rho_\mu, \rho_\nu \right] \right\} \quad \text{(3\( \rho \) coupling)} \\
&+ \frac{1}{2} g_4 \rho \left[ (-i)^2 \right] \int d^4x tr \left[ \rho_\mu, \rho_\nu \right]^2 \quad \text{(4\( \rho \) coupling)} \\
&+ i g_1 \int d^4x tr \left\{ [\alpha_\mu, \alpha_\nu] \left( \partial_\mu \rho_\nu - \partial_\nu \rho_\mu \right) \right\} \quad \text{(\( \partial \rho \)-2\( \alpha \) coupling)} \\
&+ g_2 \int d^4x tr \left\{ [\alpha_\mu, \alpha_\nu] \left[ \rho_\mu, \rho_\nu \right] \right\} \quad \text{(2\( \rho \)-2\( \alpha \) coupling)} \\
&+ g_3 \int d^4x tr \left\{ [\alpha_\mu, \alpha_\nu] \left[ \beta_\mu, \beta_\nu \right] + \left[ \rho_\mu, \beta_\nu \right] \right\} \quad \text{\( \rho \)-2\( \alpha \)-\( \beta \) coupling)} \\
&- i g_4 \int d^4x tr \left\{ \left( \partial_\mu \rho_\nu - \partial_\nu \rho_\mu \right) \left[ \beta_\mu, \rho_\nu \right] + \left[ \rho_\mu, \beta_\nu \right] \right\} \quad \text{(\( \rho \)-\( \partial \rho \)-\( \beta \) coupling)} \\
&- g_5 \int d^4x tr \left\{ \left[ \rho_\mu, \rho_\nu \right] \left[ \beta_\mu, \rho_\nu \right] + \left[ \rho_\mu, \beta_\nu \right] \right\} \quad \text{(3\( \rho \)-\( \beta \) coupling)} \\
&- \frac{1}{2} g_6 \int d^4x tr \left\{ \left[ \alpha_\mu, \rho_\nu \right] + \left[ \rho_\mu, \alpha_\nu \right] \right\}^2 \quad \text{(2\( \rho \)-2\( \alpha \) coupling)} \\
&- \frac{1}{2} g_7 \int d^4x tr \left\{ \left[ \beta_\mu, \rho_\nu \right] + \left[ \rho_\mu, \beta_\nu \right] \right\}^2 \quad \text{(2\( \rho \)-2\( \beta \) coupling)}.
\end{align*}
\]

Holographic QCD has just two parameters.

- Pion decay constant: \( f_\pi = 92.4 \text{ MeV} \)
- \( \rho \)-meson mass: \( m_\rho = 776.0 \text{ MeV} \)

All the couplings are uniquely determined!!!
Baryon as Chiral Soliton

Hedgehog soliton with B=1

\[ \pi; \quad U^*(x) = e^{i\tau_a \hat{x}_a F(r)} \]

\[ \rho; \quad \rho_0^*(x) = 0, \quad \rho_i^*(x) = \rho_{ia}(x) \tau_a = \{ \varepsilon_{iab} \hat{x}_b \tilde{G}(r) \} \tau_a \]

* Stable hedgehog soliton solution exists !!!
* Pion fields are attracted by \( \rho \)-mesons in the core of the baryon. \( \cdots \cdots \text{Shrinkage of baryon.} \)

\( \tilde{G}(r) \)

\[ F(0) = \pi \]

\[ \tilde{G}(r) \]
Baryonic matter in holographic QCD

Baryonic matter with large $N_c$.

- **Mass**: $M_{\text{baryon}} \sim O(N_c)$
  
  
  - [E.Witten, Nucl. Phys. B160, 57 (1979)]

- **Kinetic energy**:
  
  \[ \frac{p^2}{2M_{\text{baryon}}} \sim O(N_c^{-1}) \]

- **Quantum effect**
  
  - zero-point quantum fluctuation: $E_0 \sim O(N_c^0)$
  
  - $N - \Delta$ splitting: $M_{\Delta} - M_N \sim O(N_c^{-1})$

  - [G.S.Adkins et al., Nucl. Phys. B228, 552 (1983)]

**large $N_c$**

Static Skyrme matter


Skyrmion $\rightarrow R^3$
One Skyrmion on $S^3$

Static Skyrme matter

"Compactification" of one cell

One Skyrmion on $S^3$

"Delocalization" of Skyrmion

$R \rightarrow \text{small}$

$(\rho_B \rightarrow \text{large})$

$\rho_B = \frac{1}{2\pi^2 R^3}$

- deconfinement
- chiral restoration: $\langle \sigma \rangle_{\text{space}} = 0$


[H. Forkel et al., Nucl. Phys. A 504, 818 (1989)]
Total energy density of a baryon with radius $R$


A baryon delocalizes as the density increases.

Delocalization is delayed by $\rho$ mesons in the deeper interior of a baryon.
A baryon delocalizes as the density increases.

Delocalization is delayed by $\rho$ mesons in the deeper interior of a baryon.

All the physical quantities in holographic QCD are uniquely determined by two experimental inputs:

$$f_\pi = 92.4\text{MeV}, \quad m_\rho = 776\text{MeV}$$

Baryon number density for radius $R$:

$$\rho_B = \frac{1}{2\pi^2 R^3}$$
ρ meson contributions in high density phase


ρ mesons tend to disappear in high density phase !!!

\[ \pi \]
\[ F(r) \]
\[ R \text{ decreases.} \]
\[ \rho \]
\[ \hat{G}(r) \]

identity map
\[ F(r) = \pi - \frac{r}{R} \]
\[ \hat{G}(r) = 0 \]
\( \rho \) meson contributions in high density phase


(conjecture)

All the (axial) vector mesons \( B_{\mu}^{(n)} \) (\( \rightarrow \rho, a_1, \rho', a_1', \ldots \)) may tend to disappear in high density phase.

1) **Kinetic term** suppresses the spacial distribution for small \( R \).

\[
\frac{1}{2} \left( \partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)} \right)^2 \propto \frac{1}{R^2}
\]

2) **Mass term** suppresses the absolute value of \( B_{\mu}^{(n)} \) for large mass.

\[
m_{(n)}^2 \text{tr}(B_{\mu}^{(n)} B_{\mu}^{(n)})
\]

3) **Couplings** between pions and heavier mesons \( B_{\mu}^{(n)} \) become smaller.

[D.T.Son and M.A.Stephanov, PRD69, 065020(2004)]

Only pions become essential near \( \rho_C \) !!!

"pion dominance"

\[ F(0) = \pi \]
\[ F(\rho R) = 0 \]
Summary

We study baryonic matter in holographic QCD as a brane-induced Skyrmion on $S^3$.

(baryonic matter for large $N_c$)

\* Delocalization of a baryon is delayed by $\rho$ mesons in the deeper interior of it.

\* Critical density for $f_\pi = 92.4\,\text{MeV}$, $m_\rho = 776\,\text{MeV}$;

\[
\rho_c^{\text{BIS}} \approx 7.12 \rho_0 \quad \text{c.f.} \quad \rho_c^{\text{SS}} \approx 4.26 \rho_0
\]

\* Transition into uniform phase is somehow related with deconfinement and/or chiral restoration.


\* $\rho$ mesons tend to disappear in high density phase.

Generalization to all the (axial) vector mesons ($\rho, a_1, \rho', a_1', \cdots$).

Only pions become essential near $\rho_c$ !!!

“pion dominance”
Chiral order parameter

Linear: \[ \langle \sigma(x_\mu)^2 + \Pi(x_\mu)^2 \rangle^* = 0 \]

Non-linear: \[ \sigma(x_\mu)^2 + \Pi(x_\mu)^2 = f_\pi^2 \]

\[ \langle \sigma(x_\mu) \rangle^2 + \langle \Pi(x_\mu) \rangle^2 \rangle / f_\pi^2 \]

(H.Forkel et al., Nucl. Phys. A504, 818 (1989))

(Spatially-averaged chiral condensate with radius R)

\[ \rho_B : \text{large} \]
Localized order parameter

$$\Phi(R) \equiv \frac{1}{2} \int_{S^3} d^3 x \left| \bar{\varepsilon}(x) - \bar{\varepsilon}_{id} \right|$$

[Spatial fluctuation of energy density with radius $R$]

[Skyrme, BIS]

[ρ_B : large]
Gauge theory on D-branes

- Superstring theory has 10 dim. space-time to avoid ‘anomaly’.
- D-brane is introduced as fixed edges of open strings. ('D' comes from 'Dirichlet boundary condition')

\[ \Phi_{i=(p+1)\sim 9}; \text{ scalar (NG) field.} \]
\[ A_{\mu=0\sim p}; \text{ gauge field.} \]

\((p+1)\text{ dim.} \) gauge theory appears ‘on’ Dp-brane !!!
Holography

$D_p$-brane $\times N_c$

D-brane = black hole (black brane)

$([p+1]+1)$ dim. super gravity

$\lambda \to \infty$ ; strong coupling

$N_c \to \infty$

$g_s \to 0$ ; weak coupling

$\ell_s \to 0$ ; low energy

Strong-weak duality (S-duality)

1 space dim. with finite curvature.

[Polchinski (1995)]

[Maldacena (1997)]