Baryons and baryonic matter in holographic QCD

[K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D**75**, 086003 (2007)]

[K. Nawa, H. Suganuma, and T. Kojo, *to be published in* Phys. Rev. D**79** (2009)]

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INTRODUCTION

In 2005,
T. Sakai and S. Sugimoto succeeded in constructing massless QCD on D4/D8/$\overline{\text{D8}}$ multi-D brane system in type IIA superstring theory.

[T.Sakai and S.Sugimoto, Prog. Theor. Phys. 113, 843 (2005)]

"unified meson theory"

- Meson spectra of (axial) vector mesons.
- Hidden local symmetry. [Bando, et al.,1985]
- Vector meson dominance. [Sakurai, 1969]
- KSRF relation. [Kawarabayashi, Suzuki, Riazuddin, and Fayyazuddin, 1966]
- GSW model. [GellMann, Sharp, and Wegner, 1962]

- Classical supergravity is dual with large-Nc QCD.
- As a general property of large-Nc QCD, a baryon does not directly appear as a dynamical degrees of freedom.

[G.'tHooft, Nucl. Phys. B72, 461(1974); B75, 461(1974)]

*How to describe **baryons** in the large-N_c holographic model* **?**

* ***Chiral soliton picture :*** baryon as **a soliton** of meson field theory induced by holographic QCD (**"brane-induced Skyrmion"**).

* **Baryonic matter** as **a brane-induced Skyrmion on** S^3 .

CONTENTS

* Holographic QCD
(Sakai-Sugimoto model)

* Baryons in holographic QCD

[K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D**75**, 086003 (2007), hep-th/0612187]

* Baryonic matter in holographic QCD

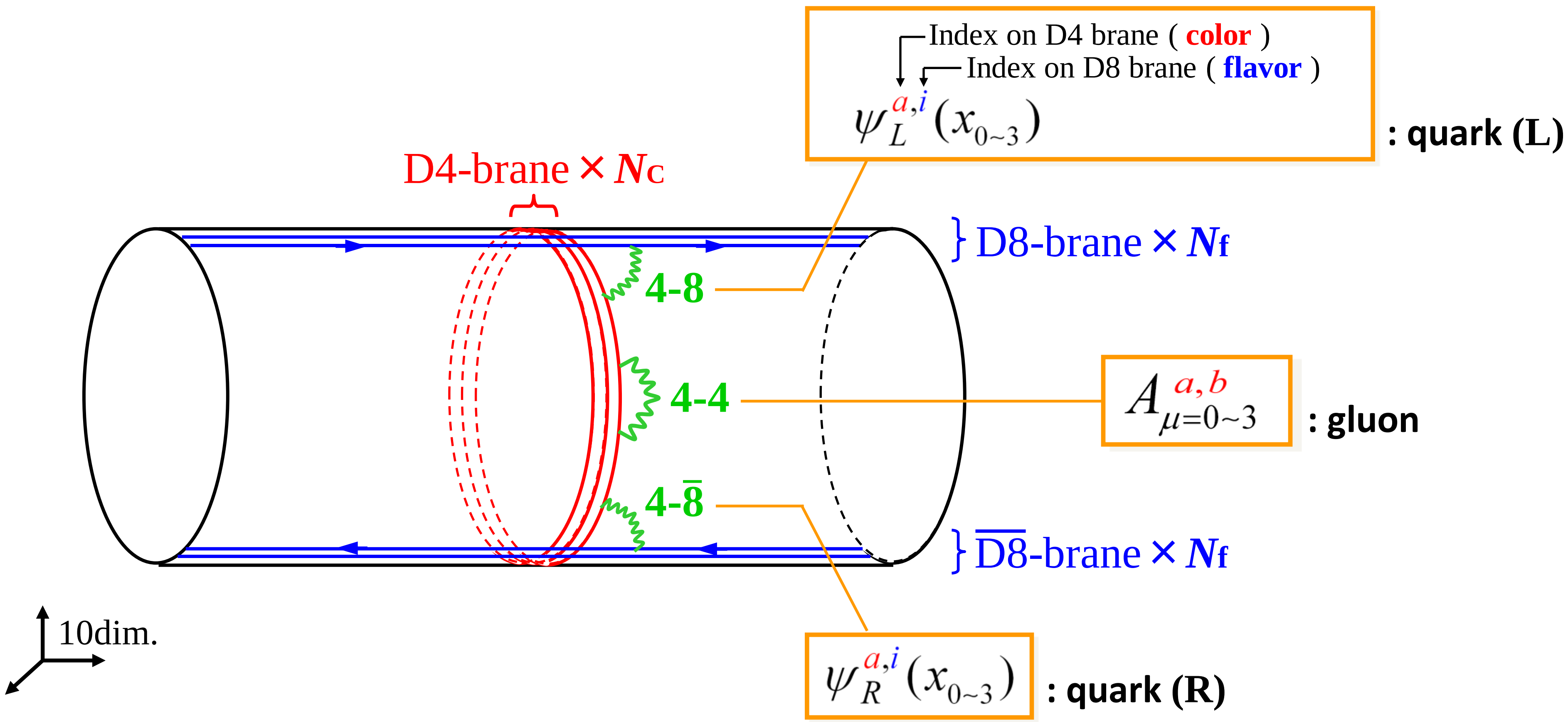
[K. Nawa, H. Suganuma, and T. Kojo, *to be published in* Phys. Rev. D**79** (2009), arXiv:0810.1005 [hep-th]]

Holographic QCD

[T.Sakai and S.Sugimoto, Prog. Theor. Phys. 113, 843 (2005)]

Construction of **massless QCD (quarks and gluons)**

on **D4/D8/$\overline{\text{D8}}$-branes.**

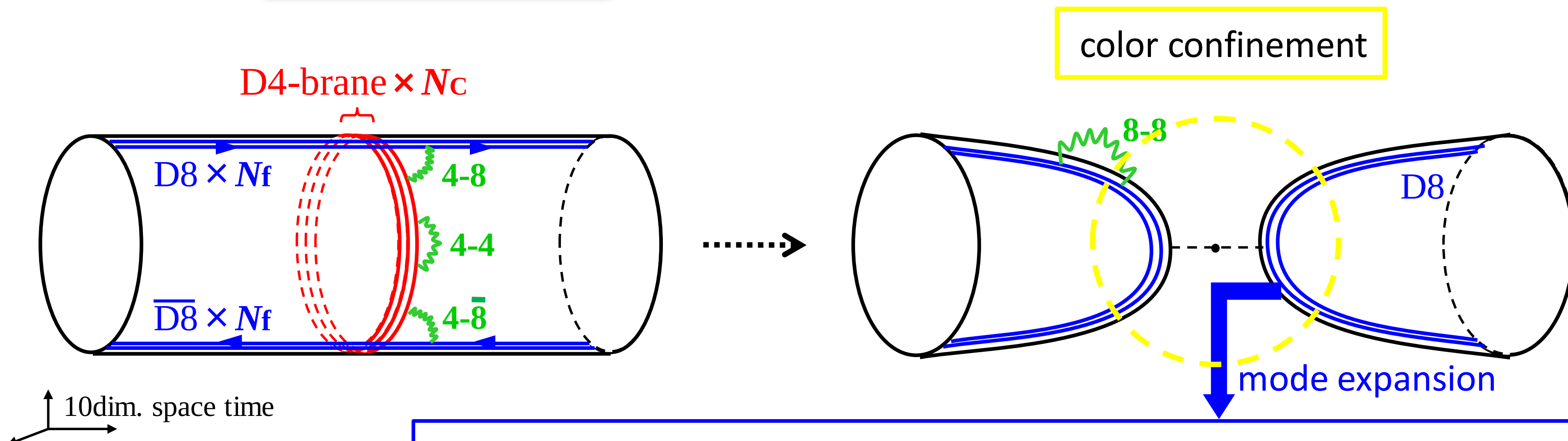


Probe approximation with $N_c >> N_f$

Mass of D*p*-brane is proportional to its **sheets number**.

D4 × *N*c → SUGRA background

D8, $\overline{\text{D8}}$ × *N*f → "probe"



$$A_\mu(x_\nu,z)=\frac{1}{i}\xi^{-1}(x_\nu)\partial_\mu\xi(x_\nu)\Psi_+(z)+\frac{1}{i}\xi(x_\nu)\partial_\mu\xi^{-1}(x_\nu)\Psi_-(z)+\sum_{n\geq1}B_\mu^{(n)}(x_\nu)\Psi_n(z)$$

pion

(axial) vector mesons

$\rho, a_1, \rho', a_1', \rho'', \cdots$

Baryon does not directly appear as a general property of large-Nc QCD. **[G.'tHooft, Nucl. Phys. B72, 461(1974); B75, 461(1974)]**

Baryons in holographic QCD

∗ chiral perturbation theory (chiral symmetry × Lorentz invariance)

2-derivative term

$\mathcal{L}_2$: $\mathrm{tr}(L_\mu L_\mu)$

4-derivative term

$\mathcal{L}_4$: $\mathrm{tr}[L_\mu, L_\nu]^2$, ~~$\mathrm{tr}\{L_\mu, L_\nu\}^2$~~ , ~~$\mathrm{tr}(\partial_\mu L_\mu)^2$~~

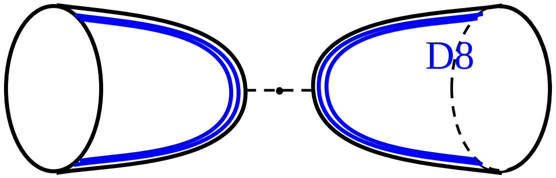
Instability of Skyrme soliton.

[I. Zahed and G.E. Brown, Phys. Rep. 142, 1 (1986)]

Topological picture for baryon is also supported by superstring theory.

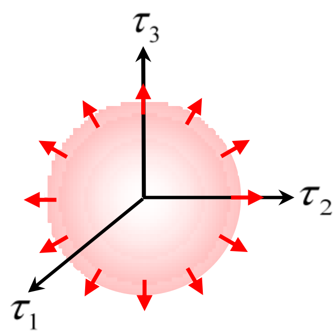
Effective action of D8 brane

$$\kappa \int d^4x dz \mathrm{tr} \left(\frac{1}{2} K^{-1/3} F_{\mu\nu} F_{\mu\nu} + K F_{\mu z} F_{\mu z} \right)$$



Baryon as a chiral soliton

[K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D**75**, 086003 (2007)]



c.f. ***Baryon as an instanton in 5dim. gauge theory on D8.***

[D.K.Hong, M.Rho,H.-U.Yee, and P.Yi, PRD76, 061901(2007)]

[H.Hata, T.Sakai, S.Sugimoto, and S. Yamato, PTP117,1157(2007)]

4dim. meson effective action from holographic QCD (pions and ρ-mesons)

pion field :
$\xi(x_\mu) = e^{i\pi(x_\mu)/f_\pi}$

1-form of pion field :
$L_\mu = \frac{1}{i} U^\dagger \partial_\mu U = \frac{1}{i} \xi^{2\dagger} \partial_\mu \xi^2$

axial vector current :
$\alpha_\mu = \frac{1}{i} \xi^{-1} \partial_\mu \xi - \frac{1}{i} \xi \partial_\mu \xi^{-1}$

vector current :
$\beta_\mu = \frac{1}{2}\left(\frac{1}{i} \xi^{-1} \partial_\mu \xi + \frac{1}{i} \xi \partial_\mu \xi^{-1} \right)$

Holographic QCD has just **two parameters**.

- pion decay constant : $f_\pi = 92.4\text{MeV}$
- ρ-meson mass : $m_\rho = 776.0\text{MeV}$

➡ All the couplings are **uniquely determined** !!!

[K. Nawa et al., Phys. Rev. D**75**, 086003 (2007)]

$$
\begin{aligned}
& S^{DBI}_{D8} - S^{DBI}_{D8}|_{A_M \to 0} \\
&= \kappa \int d^4x dz \, \mathrm{tr} \left\{ \frac{1}{2} K(z)^{-\frac{1}{3}} F_{\mu\nu} F_{\mu\nu} + K(z) F_{\mu z} F_{\mu z} \right\} \\
&= \frac{f_\pi^2}{4} \int d^4x \, \mathrm{tr}\, (L_\mu L_\mu) \quad \text{(chiral term)} \\
&+ \frac{1}{2} \frac{1}{16 e^2} [i^2] \int d^4x \, \mathrm{tr}\, [L_\mu, L_\nu]^2 \quad \text{(Skyrme term)} \\
&+ \frac{1}{2} \int d^4x \, \mathrm{tr}\, (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)^2 \quad \text{(ρ-kinetic term)} \\
&+ m_\rho^2 \int d^4x \, \mathrm{tr}\, (\rho_\mu \rho_\mu) \quad \text{(ρ-mass term)} \\
&+ \frac{1}{2} 2 g_{3\rho} [-i] \int d^4x \, \mathrm{tr} \left\{ (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) [\rho_\mu, \rho_\nu] \right\} \quad \text{(3ρ coupling)} \\
&+ \frac{1}{2} g_{4\rho} [(-i)^2] \int d^4x \, \mathrm{tr}\, [\rho_\mu, \rho_\nu]^2 \quad \text{(4ρ coupling)} \\
&+ i \, g_1 \int d^4x \, \mathrm{tr} \left\{ [\alpha_\mu, \alpha_\nu] (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) \right\} \quad \text{($\partial\rho$-2α coupling)} \\
&+ g_2 \int d^4x \, \mathrm{tr} \left\{ [\alpha_\mu, \alpha_\nu] [\rho_\mu, \rho_\nu] \right\} \quad \text{(2ρ-2α coupling)} \\
&+ g_3 \int d^4x \, \mathrm{tr} \left\{ [\alpha_\mu, \alpha_\nu] ([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu]) \right\} \quad \text{(ρ-2α-β coupling)} \\
&- i \, g_4 \int d^4x \, \mathrm{tr} \left\{ (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) ([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu]) \right\} \quad \text{(ρ-$\partial\rho$-β coupling)} \\
&- g_5 \int d^4x \, \mathrm{tr} \left\{ [\rho_\mu, \rho_\nu] ([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu]) \right\} \quad \text{(3ρ-β coupling)} \\
&- \frac{1}{2} g_6 \int d^4x \, \mathrm{tr}\, ([\alpha_\mu, \rho_\nu] + [\rho_\mu, \alpha_\nu])^2 \quad \text{(2ρ-2α coupling)} \\
&- \frac{1}{2} g_7 \int d^4x \, \mathrm{tr}\, ([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])^2 \quad \text{(2ρ-2β coupling).}
\end{aligned}
$$

Baryon as Chiral Soliton

[K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D**75**, 086003 (2007)]

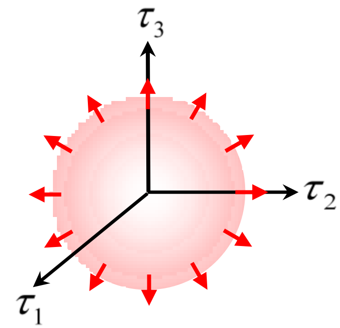
Hedgehog soliton with B=1

[T.H.R. Skyrme, Proc. R. Soc. A260, 127 (1961)]

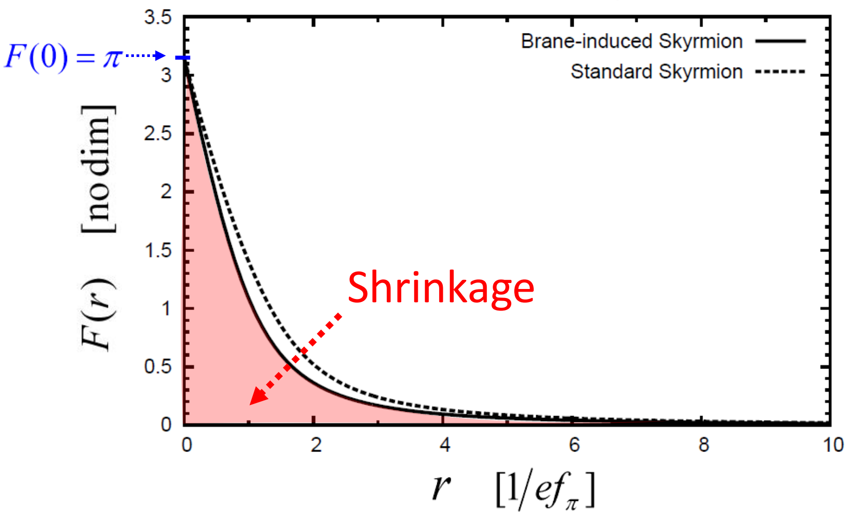
π ; $U^\star(\mathbf{x}) = e^{i\tau_a \hat{x}_a F(r)}$

ρ ; $\rho_0^\star(\mathbf{x}) = 0, \quad \rho_i^\star(\mathbf{x}) = \rho_{ia}^\star(\mathbf{x})\tau_a = \left\{\varepsilon_{iab}\hat{x}_b \tilde{G}(r)\right\}\tau_a$

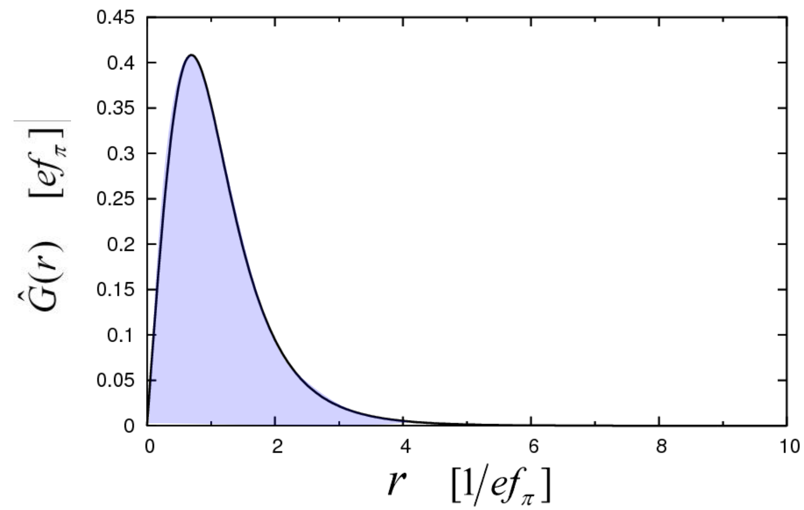
Pauli matrix



(pion)



(ρ-meson)



* Stable hedgehog soliton solution exists !!!

* Pion fields are attracted by ρ-mesons in the core of the baryon. ······▸ ***Shrinkage of baryon.***

c.f. ***Skyrmions with HLS.***

[Y.Igarashi et al., Nucl. Phys. B 259, 721 (1985)]

Strong repulsion for baryon.

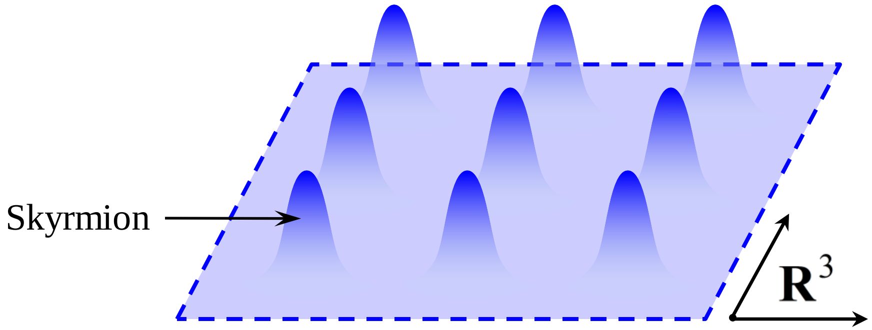
Baryonic matter in holographic QCD

Baryonic matter with large *N*c.

- Mass : $M_{\text{baryon}} \sim O(N_C)$ [G.'tHooft, Nucl. Phys. B72, 461(1974); B75, 461(1974)] [E.Witten, Nucl. Phys. B160, 57 (1979)]
- Kinetic energy : $\dfrac{\mathbf{p}^2}{2M_{\text{baryon}}} \sim O(N_C^{-1})$
- Quantum effect
  - zero-point quantum fluctuation : $E_0 \sim O(N_C^0)$
  - $N-\Delta$ splitting : $M_\Delta - M_N \sim O(N_C^{-1})$ [G.S.Adkins et al., Nucl. Phys. B228, 552 (1983)]

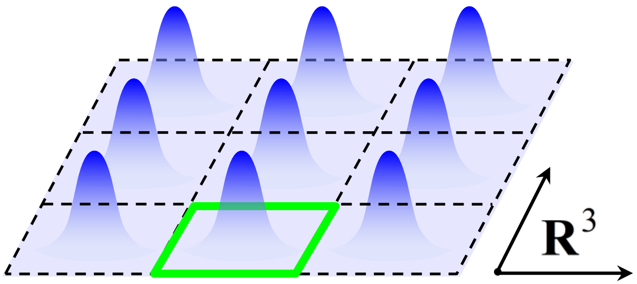
large *N*c

Static Skyrme matter [I.Klebanov, Nucl. Phys. B262, 133 (1985)]



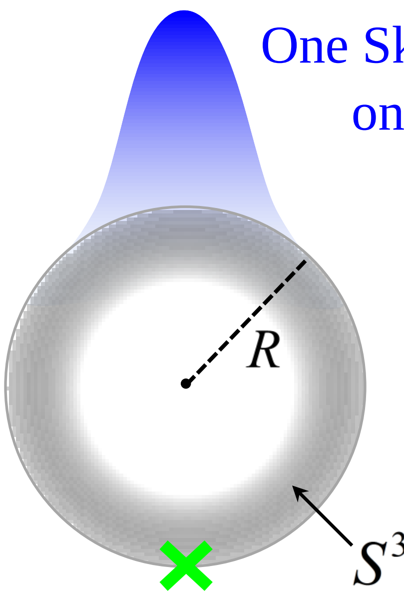
One Skyrmion on S^3

Static Skyrme matter



"Compactification" of one cell

One Skyrmion on S^3

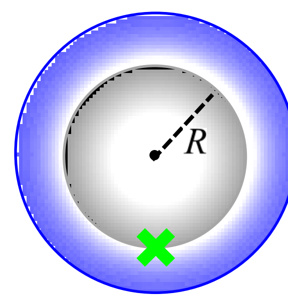


$R \longrightarrow$ *small*

($\rho_B \longrightarrow$ *large*)

$$\rho_B = \frac{1}{2\pi^2 R^3}$$

"**Delocalization**" of Skyrmion



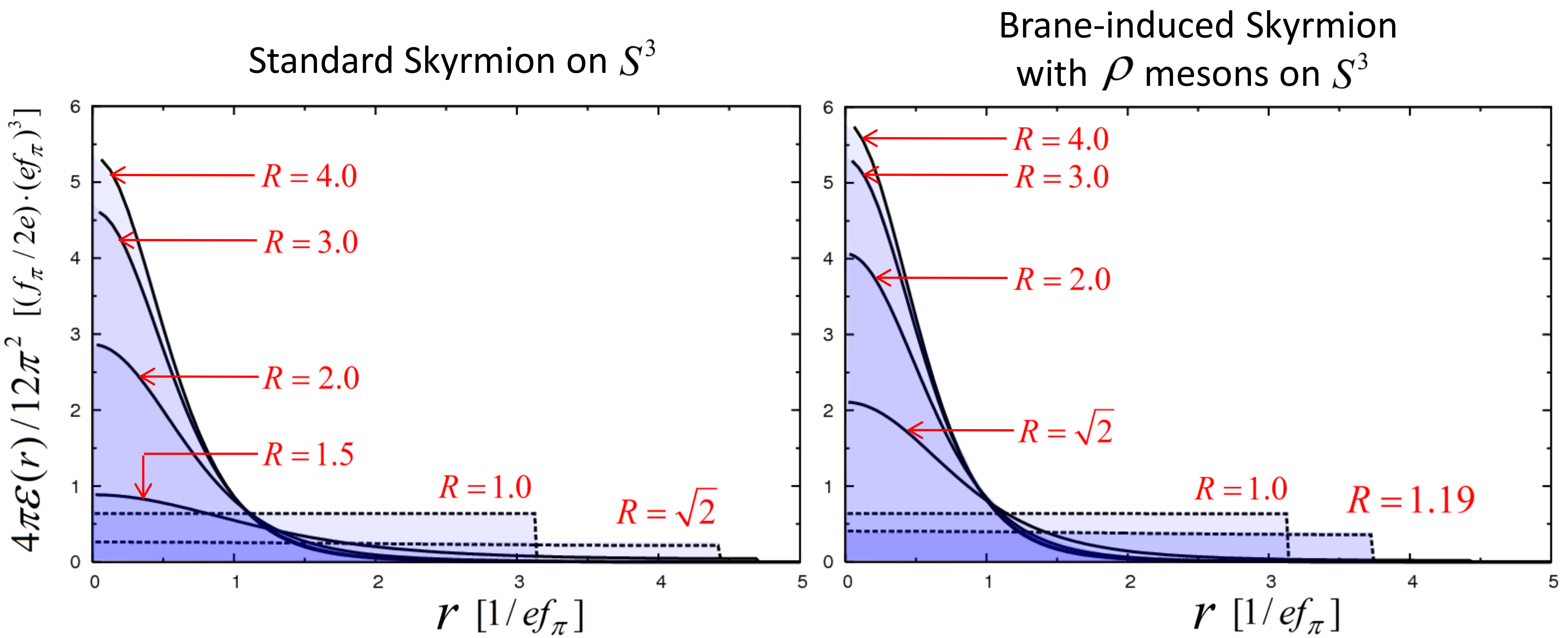
[A.D. Jackson et al., Nucl. Phys.A **486**, 634 (1998)].
[H.Forkel et al., Nucl. Phys. A**504**, 818 (1989)]

- deconfinement
- chiral restoration : $\langle \sigma \rangle_{\text{space}} = \mathbf{0}$

[N.S.Manton and P.J.Ruback, Phys. Lett. B181, 137 (1986)]

Total energy density of a baryon with radius R

[K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D**79** (*to appear*)(2009)]

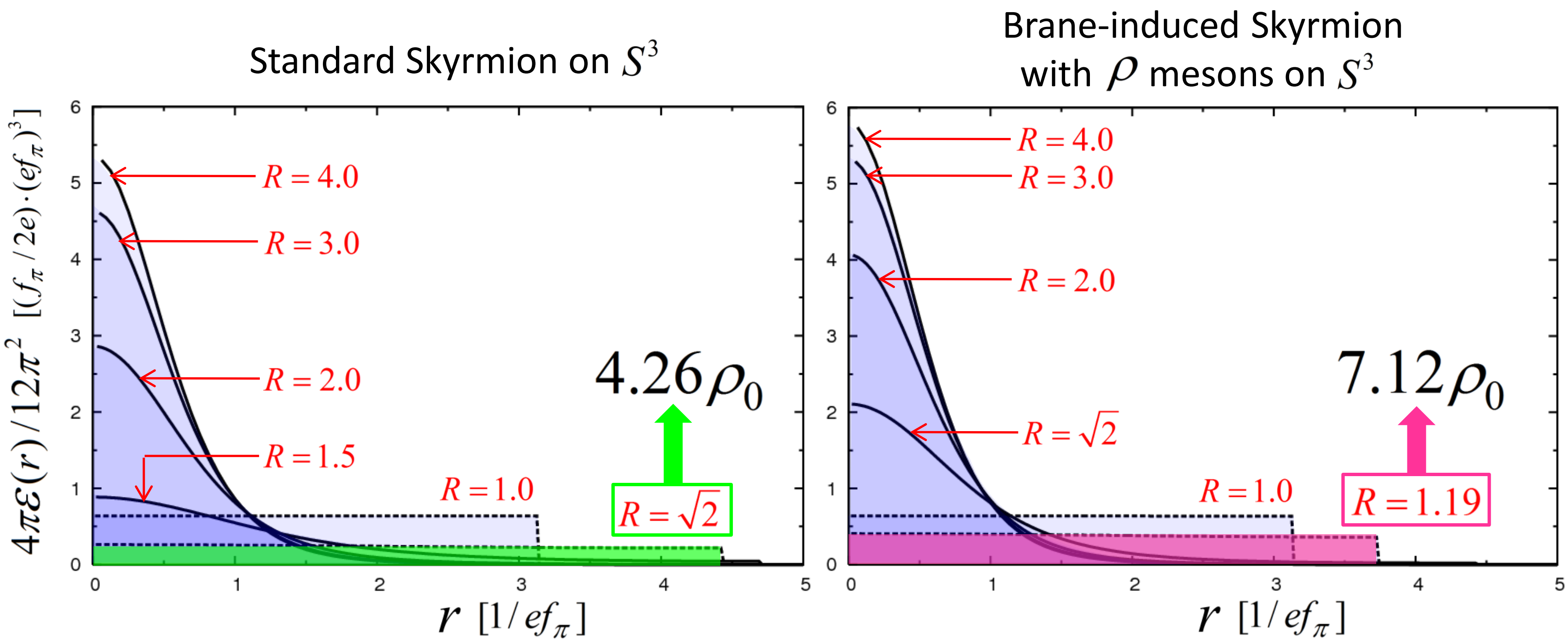


* A baryon delocalizes as the density increases.

* Delocalization is delayed by ρ mesons in the deeper interior of a baryon.

Total energy density of a baryon with radius R

[K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D**79** (*to appear*)(2009)]



* A baryon delocalizes as the density increases.

* Delocalization is delayed by ρ mesons in the deeper interior of a baryon.

- All the physical quantities in holographic QCD are uniquely determined by **two** experimental inputs:

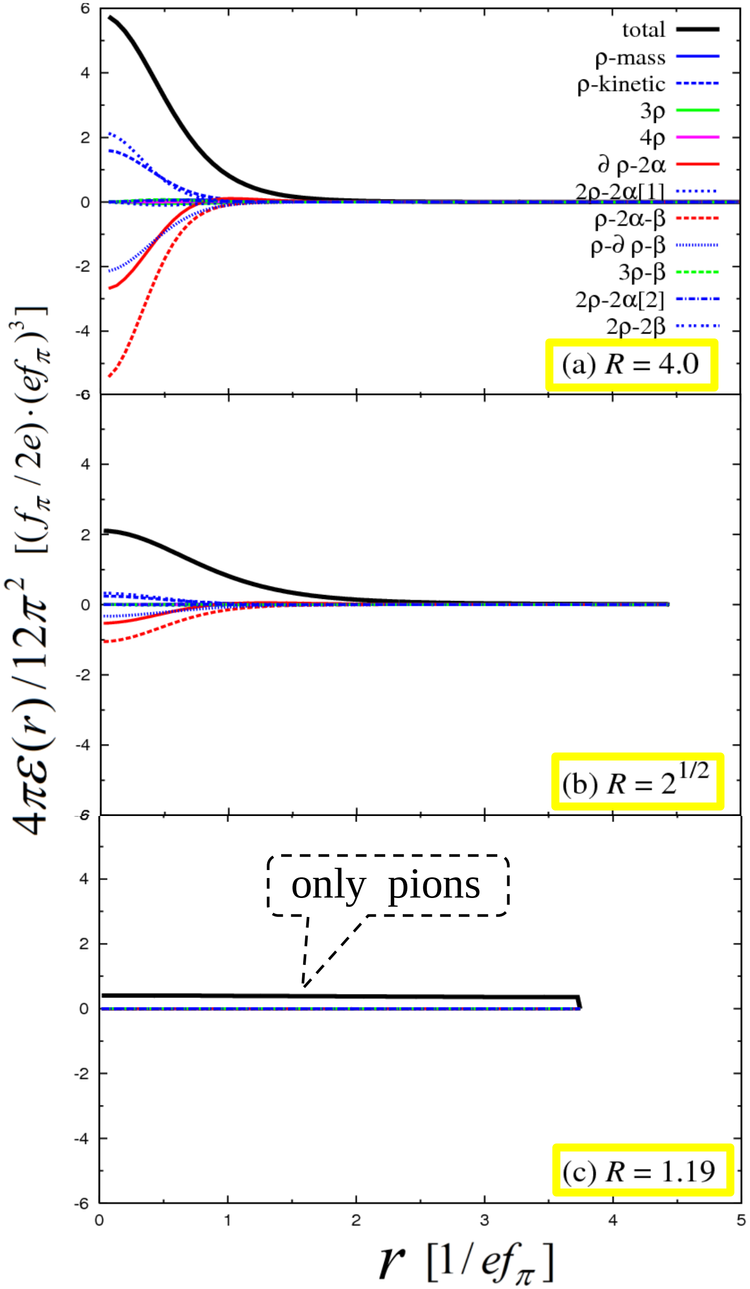
$$f_\pi = 92.4\text{MeV},\ m_\rho = 776\text{MeV}$$

- Baryon number density for radius ***R*** :

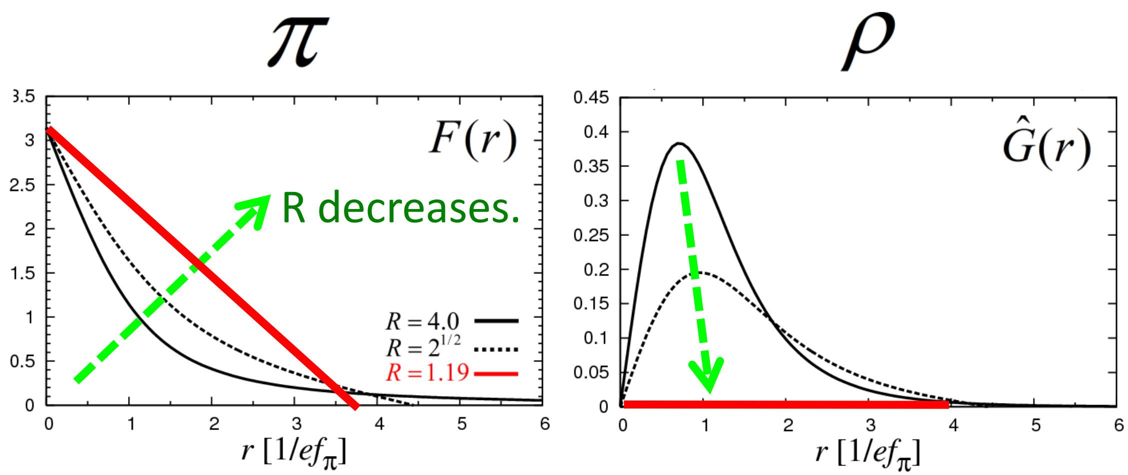
$$\rho_B = \frac{1}{2\pi^2 R^3}$$

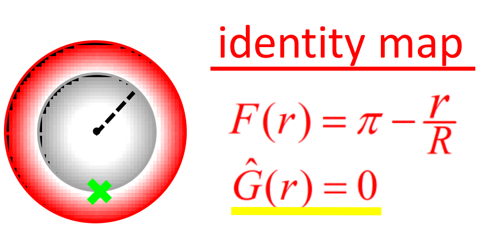
ρ meson contributions in high density phase

[K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D**79** (*to appear*)(2009)]

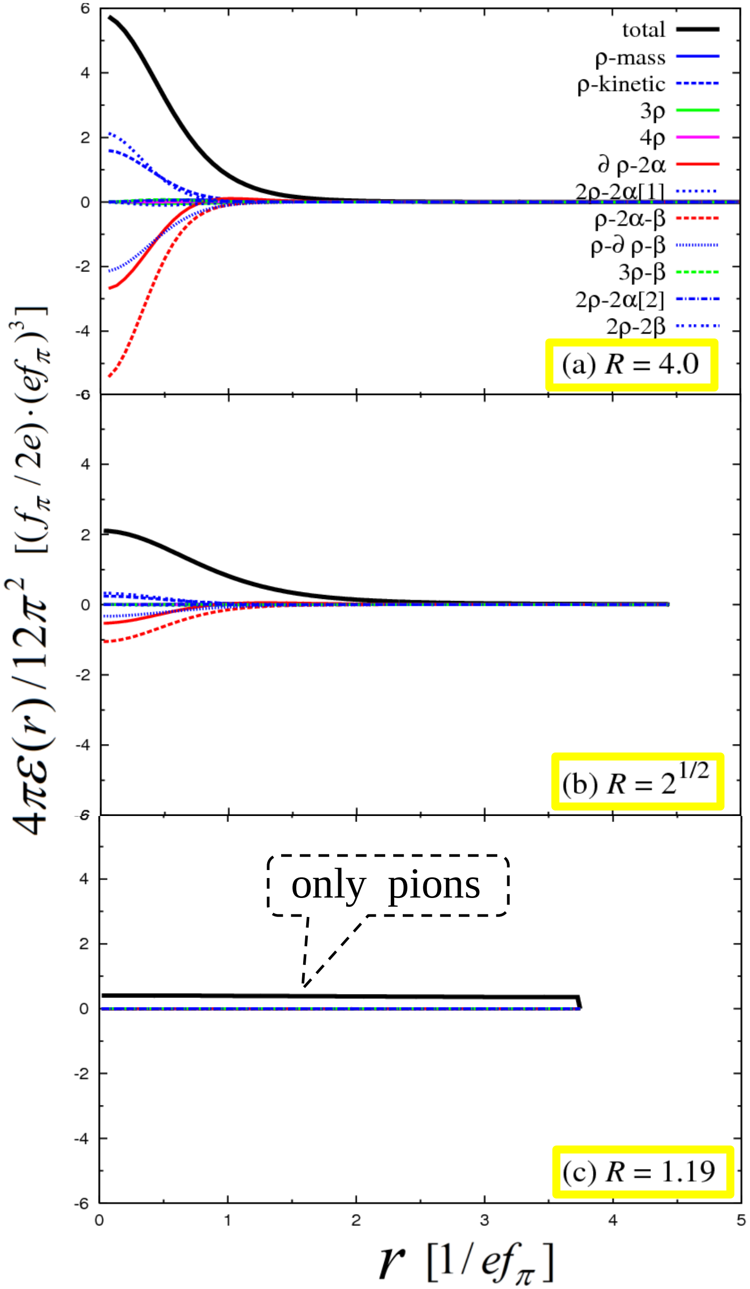


ρ mesons tend to disappear in high density phase !!!





ρ meson contributions in high density phase



[K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D**79** (*to appear*)(2009)]

(conjecture)

All the (axial) vector mesons $B_\mu^{(n)}$ $(\to \rho, a_1, \rho', a_1', \cdots)$ *may* tend to disappear in high density phase.

1) Kinetic term suppresses the spacial distribution for small R.

$$\frac{1}{2}(\partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)})^2 \propto 1/R^2$$

2) Mass term suppresses the absolute value of $B_\mu^{(n)}$ for large mass.

$$m_{(n)}^2 \mathrm{tr}(B_\mu^{(n)} B_\mu^{(n)})$$

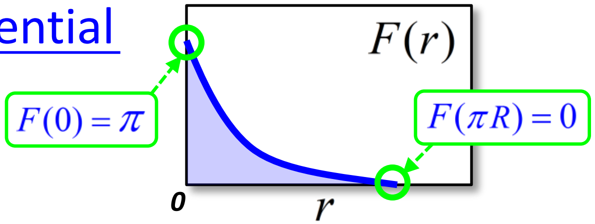
3) Couplings between pions and heavier mesons $B_\mu^{(n)}$ become smaller.

[D.T.Son and M.A.Stephanov, PRD69, 065020(2004)]

[K. Nawa, H. Suganuma, and T. Kojo,PRD75, 086003 (2007)]

Only pions become essential near ρ_c !!!

"pion dominance"



Summary

We study baryonic matter in holographic QCD as a brane-induced Skyrmion on S^3.

(*baryonic matter for* ***large Nc***)

* **Delocalization** of a baryon **is delayed** by ρ mesons in the deeper interior of it.

- Critical density for $f_\pi = 92.4\text{MeV}$, $m_\rho = 776\text{MeV}$;

$$\rho_c^{\text{BIS}} \simeq 7.12\rho_0 \quad \text{c.f.} \quad \rho_c^{\text{SS}} \simeq 4.26\rho_0$$

- Transition into uniform phase is somehow related with deconfinement and/or chiral restoration.

[N.S.Manton and P.J.Ruback, Phys. Lett. B181, 137 (1986)]

* ρ mesons tend to **disappear** in high density phase.

conjecture → Generalization to all the (axial) vector mesons ($\rho, a_1, \rho', a_1', \cdots$).

Only pions become essential near ρ_c !!!

"**pion dominance**"

Chiral order parameter

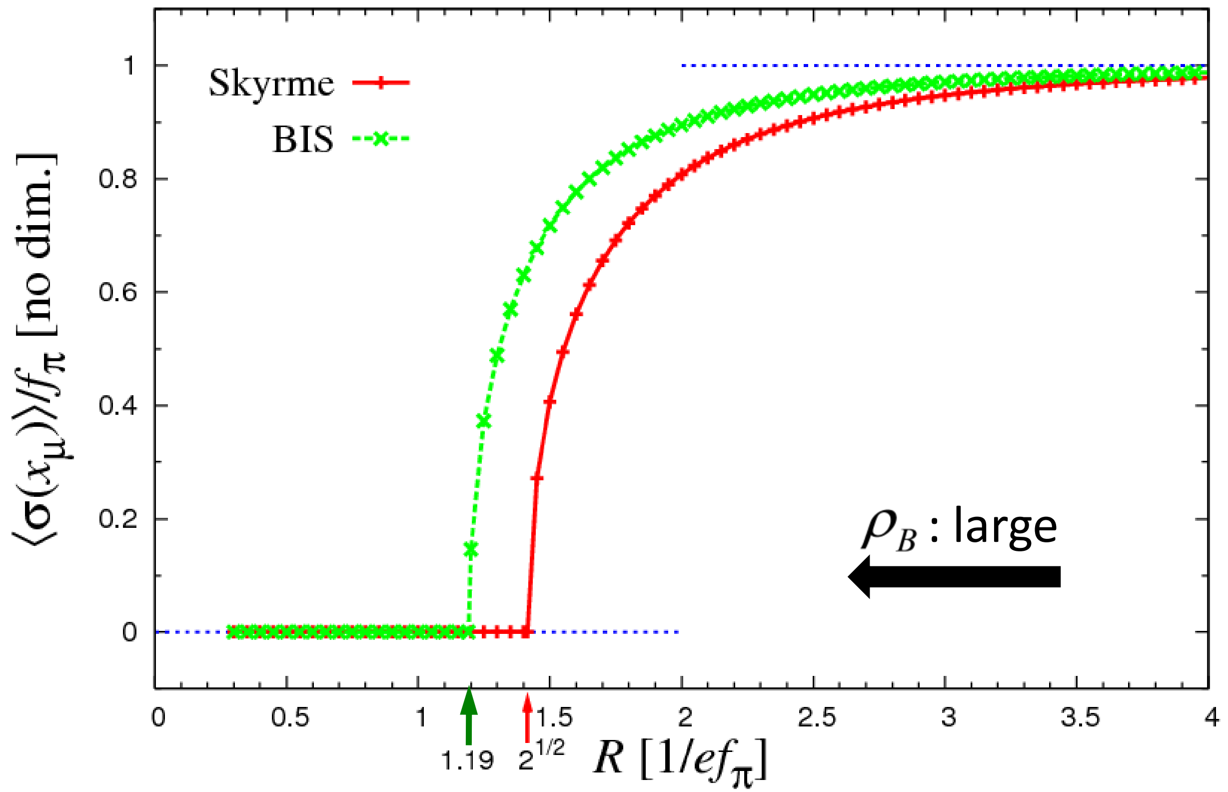
Linear : $\langle \sigma(x_\mu)^2 + \mathbf{\Pi}(x_\mu)^2 \rangle^* = 0$

Non-linear : $\sigma(x_\mu)^2 + \mathbf{\Pi}(x_\mu)^2 = f_\pi^2$

$$\{\langle \sigma(x_\mu) \rangle^2 + \langle \mathbf{\Pi}(x_\mu) \rangle^2\} / f_\pi^2$$

[H.Forkel et al., Nucl. Phys. A**504**, 818 (1989)]

(Spatially-averaged chiral condensate with radius *R*)

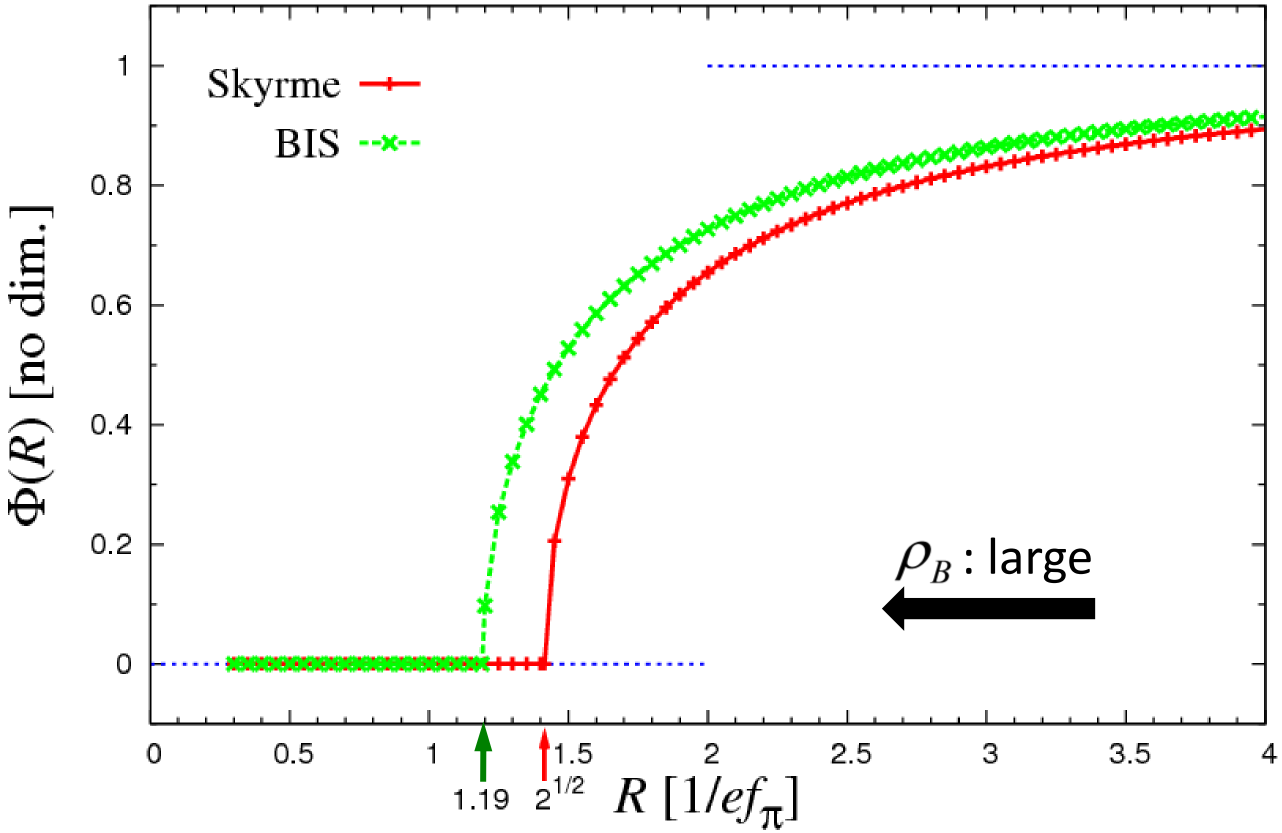


Localized order parameter

$$\Phi(R) \equiv \frac{1}{2}\int_{S^3} d^3x \left|\bar{\varepsilon}(\mathbf{x}) - \bar{\varepsilon}_{\rm id}\right|$$

[A.D. Jackson et al., Nucl. Phys.A **486**, 634 (1998)].

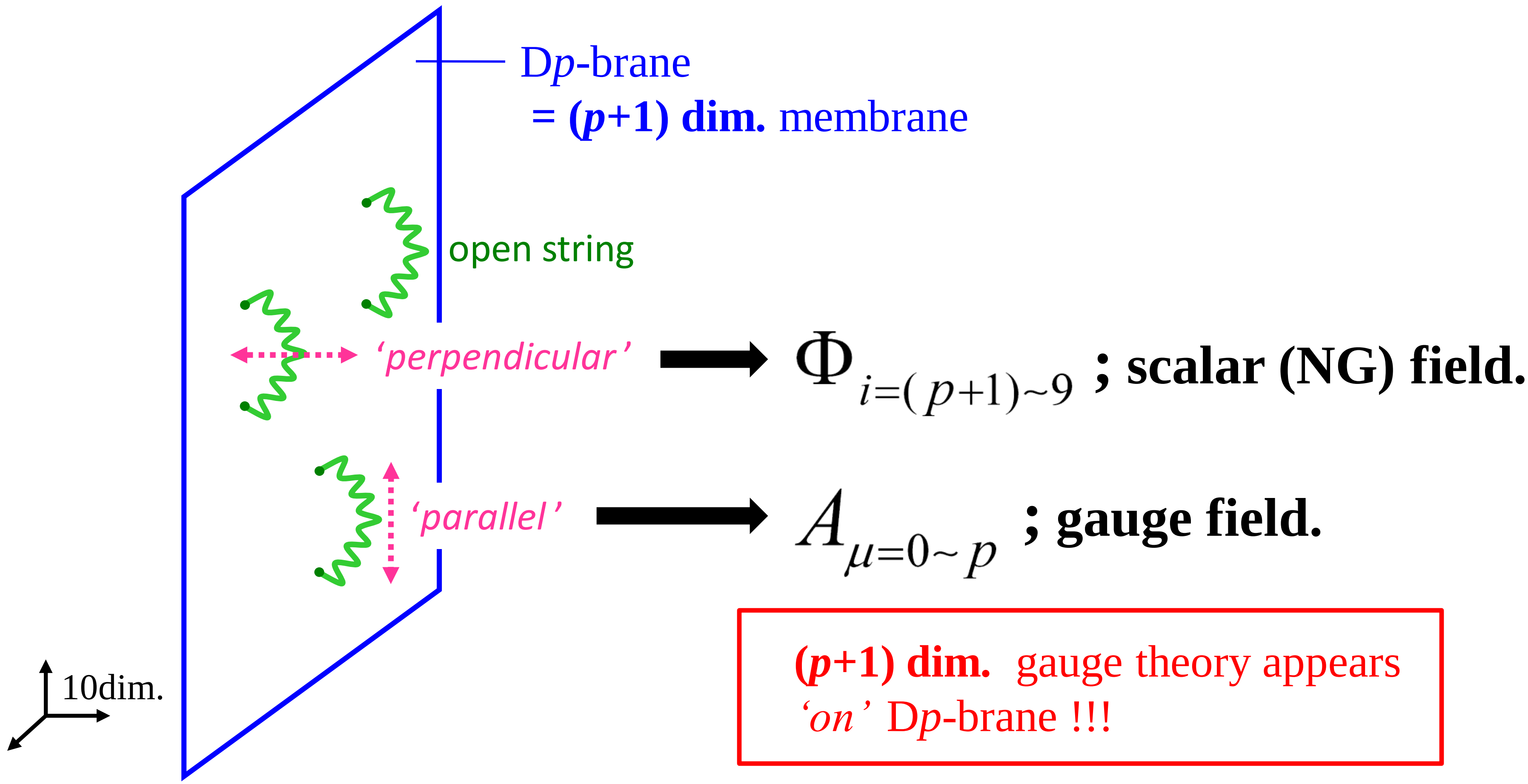
(Spatial fluctuation of energy density with radius *R*)



Gauge theory *on* D-branes

- Superstring theory has **10 dim.** space-time to avoid 'anomaly'.
- D-brane is introduced as fixed edges of open strings.

('**D**' comes from '**D**irichlet boundary condition')



(*p*+1) dim. gauge theory appears '*on*' D*p*-brane !!!

Holography

[Maldacena (1997)]