Phase Transitions of Charged Kerr-AdS Black Holes from Large N Gauge Theories

Tatsuma Nishioka

(Kyoto Univ.)

with

K. Murata, N. Tanahashi, H. Yumisaki (Kyoto U)

arXiv:0806.2314

Introduction

AdS/CFT Correspondence relates (Maldacena '98)

- Type IIB SUGRA on $AdS_5 \times S^5$
- $\mathcal{N} = 4$ Super Yang-Mills (CFT) on the boundary of AdS_5

At finite temperature (Witten '98)

- AdS Schwarzschild black hole
- Super Yang-Mills on $S^1 \times S^3$

Gravity theory is expected to correspond to the strongly-coupled gauge theory

Hawking-Page transition and AdS/CFT

The AdS/CFT relates (Witten '98)

- The Hawking-Page transition in the gravity theory
- The confinement/deconfinement transition in the gauge theory

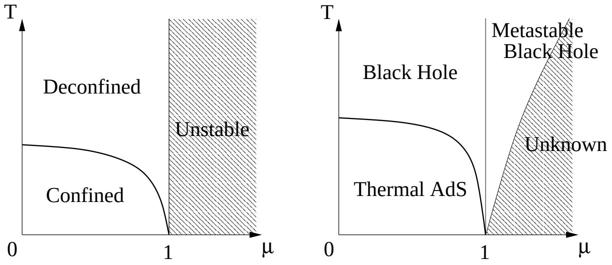
The gauge theory at finite temperature

- defined on $S^1 \times S^3$
- described as the unitary matrix model (Sundborg '99, Aharony et al. '03)
- exhibits the phase transition in the large N limit even at zero coupling

Including chemical potentials

What happens when we include chemical potentials?

- Including R-symmetry chemical potentials has been considered (Yamada-Yaffe '06)
- This corresponds to the R-charged black hole
- The resulting phase diagram of the gauge theory is similar to that of dual gravity



Our purpose

We treat most general case with

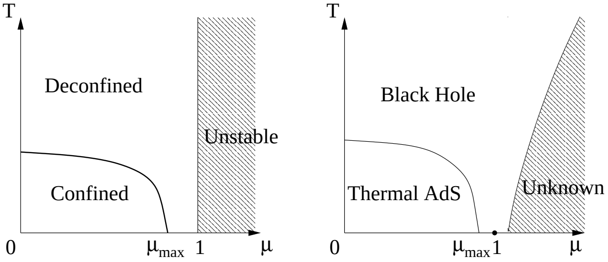
- three R-symmetry chemical potentials
- two $SO(4)$-symmetry chemical potentials associated with S^3

This is dual to charged Kerr-AdS black hole

- the solutions with four degree of freedom are known
  (Mei-Pope '07, Cvetic-Lu-Pope '04, Cvetic-Gibbons-Lu-Pope '05)
- Most general solution has not been yet discovered
- We can study the property of such a undiscovered black hole from the gauge theory side

Resulting phase diagram

- We found a gap between the confinement/deconfinement transition line and the boundary of the phase diagram in the gauge theory
- We also found such a gap in dual gravity to study the its thermodynamics and instability



$\mathcal{N} = 4$ SYM and chemical potentials

We study the $\mathcal{N} = 4$ SYM on S^3

$$I_{\mathcal{N}=4} = -\int d^4x \sqrt{-g}\, tr\Big[\frac{1}{2}(F_{\mu\nu})^2 + (D_\mu \phi_m)^2 + \phi_m^2 + i\bar{\lambda}^A \Gamma^\mu D_\mu \lambda_A - \frac{g^2}{2}[\phi_m, \phi_n]^2 - g\bar{\lambda}^A \Gamma^m [\phi_m, \lambda_A]\Big],$$

where the covariant derivative is defined by

$$D_\mu = \nabla_\mu + ig[A_\mu, \ \cdot\ ]$$

This action has

- $SO(4)$ symmetry from S^3
- $SO(6) \simeq SU(4)$ symmetry from the R-symmetry

Partition function with chemical potentials

- $SO(4)$ symmetry has two charges $\hat{J}_1, \hat{J}_2$
- $SU(4)$ symmetry has three charges $\hat{Q}_1, \hat{Q}_2, \hat{Q}_2$

The grand canonical partition function is given by

$$Z(\beta) = Tr[e^{-\beta(\hat{H} - \mu_a \hat{Q}_a - \Omega_i \hat{J}_i)}]$$

Notice that the partition function of dual gravity is written as

$$\begin{aligned} Z_{gravity} &= \exp\left[-I_{graviry}\right] \\ &= \exp\left[-\beta(E - \mu_a Q_a - \Omega_i J_i) + S\right] \end{aligned}$$

Large N gauge theory and unitary matrix model

Consider the $\mathcal{N}=4$ SYM on $S^1 \times S^3$ at zero coupling

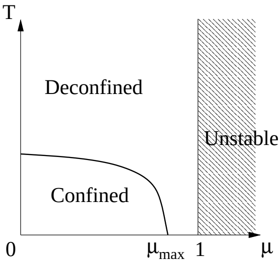
$$Z_{\mathcal{N}=4} = \int \mathcal{D}A_\mu \mathcal{D}\psi_i \mathcal{D}\phi_a \ \exp[-I_{\mathcal{N}=4}], \qquad (a=1,\dots,6, \quad i=1,\dots,4)$$

- The only temporal gauge field A_0 has zero mode α
- We must maintain Gauss' law constraint even at zero coupling
- After integrate out the massive modes

$$Z_{\mathcal{N}=4} = \int [dU] \exp[-I(U)], \qquad U \equiv e^{i\alpha}$$

Summary of the gauge theory

- We determine the maximal chemical potentials μ_{max} where the transition line ends at $T = 0$
- The gap appears when there are more than four chemical potentials
- We found the phase diagram is bounded by $\mu_a = 1, \Omega_i = 1$, above which some modes of the scalar fields become tachyonic



Charged Kerr-AdS black hole

The five dimensional charged Kerr-AdS black hole with $(J_1, J_1, Q_1, Q_2, Q_3)$ charges is defined by the following metric

$$ds^2 = -\frac{Y - f_3}{r^4 H^{2/3}} dt^2 + \frac{r^4 H^{1/3}}{Y} dr^2 + r^2 H^{1/3} d\Omega_3^2 + \frac{f_1 - r^6 H}{r^4 H^{2/3}} (\sin^2\theta d\phi_1 + \cos^2\theta d\phi_2)^2 - \frac{2f_2}{r^4 H^{2/3}} dt (\sin^2\theta d\phi_1 + \cos^2\theta d\phi_2)$$

$$A^a = \frac{2}{r^2 H_a} \left\{ s_a c_a dt + a(c_a s_b s_c - s_a c_b c_c)(\sin^2\theta d\phi_1 + \cos^2\theta d\phi_2) \right\}, \qquad d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi_1^2 + \cos^2\theta d\phi_2^2,$$

$$X_a = H_a^{-1} H^{1/3}, \qquad H = H_1 H_2 H_3, \qquad H_a = 1 + \frac{2ms_a^2}{r^2}, \qquad s_a = \sinh\delta_a, \qquad c_a = \cosh\delta_a,$$

$$f_1 = r^6 H + 2ma^2 r^2 + 4m^2 a^2 \left[2\Big(\prod_a c_a - \prod_a s_a\Big) \prod_b s_b - \sum_{a<b} s_a^2 s_b^2 \right], \qquad f_2 = 2ma\Big(\prod_a c_a - \prod_a s_a\Big) r^2 + 4m^2 a \prod_a s_a,$$

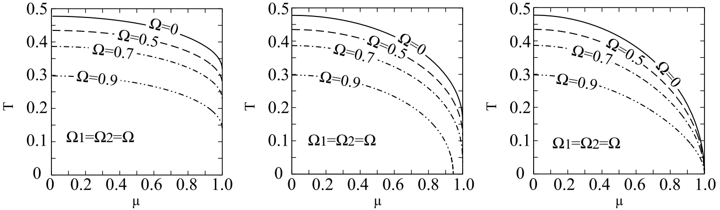
$$f_3 = 2ma^2(1 + r^2) + 4m^2 a^2 \left[2\Big(\prod_a c_a - \prod_a s_a\Big) \prod_b s_b - \sum_{a<b} s_a^2 s_b^2 \right], \qquad Y = f_3 + r^6 H + r^4 - 2mr^2$$

Phase transition in dual gravity

The action relative to pure AdS space

$$I_{gravity} = (M - TS - 2\Omega J - \mu_1 Q_1 - \mu_2 Q_2 - \mu_3 Q_3)/T \ .$$

- The Hawking-Page transition occurs at $I_{gravity} = 0$
- Below the lines ($I_{gravity} > 0$): Thermal AdS space is favored
- Above the lines ($I_{gravity} < 0$): Charged Kerr-AdS black hole is favored



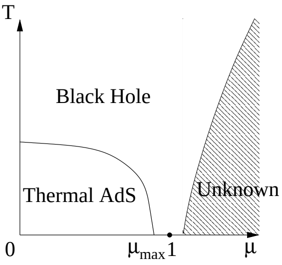
(a) $\mu_1 \equiv \mu, \mu_2 = \mu_3 = 0$ (b) $\mu_1 = \mu_2 \equiv \mu, \mu_3 = 0$ (c) $\mu_1 = \mu_2 = \mu_3 \equiv \mu$

Instability of charged Kerr-AdS black hole

- It has been suggested that this unitarity line in the gauge theory corresponds to the thermodynamical instability line on the phase diagram in the dual gravity theory

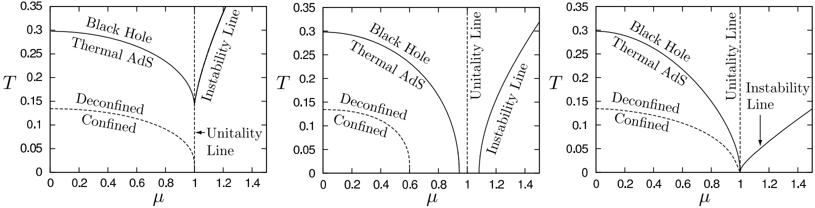
  (Yamada-Yaffe '06)

- We search such a line and obtained the below phase diagram



Comparison via AdS/CFT

- It is interesting to compare the phase diagrams beteen the gauge theory and dual gravity
- We found remarkable agreement in the phase diagrams!



(d) $\mu_1 \equiv \mu, \mu_2 = \mu_3 = 0$ (e) $\mu_1 = \mu_2 \equiv \mu, \mu_3 = 0$ (f) $\mu_1 = \mu_2 = \mu_3 \equiv \mu$

Summary

- We have constructed the gauge theory on S^3 dual to the charged Kerr-AdS black hole by adding the chemical potentials associated with $SO(4)$ and R-symmetry
- This model shows the confinement/deconfinement transition which is dual to the Hawking-Page transition in the gravity theory
- The phase diagrams have remarkable similarities in both sides
- Especially we found a new gap between the tansition line and the boundary of the phase diagram when there are more than four chemical potentials
- We have studied the ratios of the thermodynamical quantities and showed quantatively that the gauge tehory describes the dual gravity even near the critical temperature