Various Faces of Extreme QCD

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December 2008 at Kyusyu

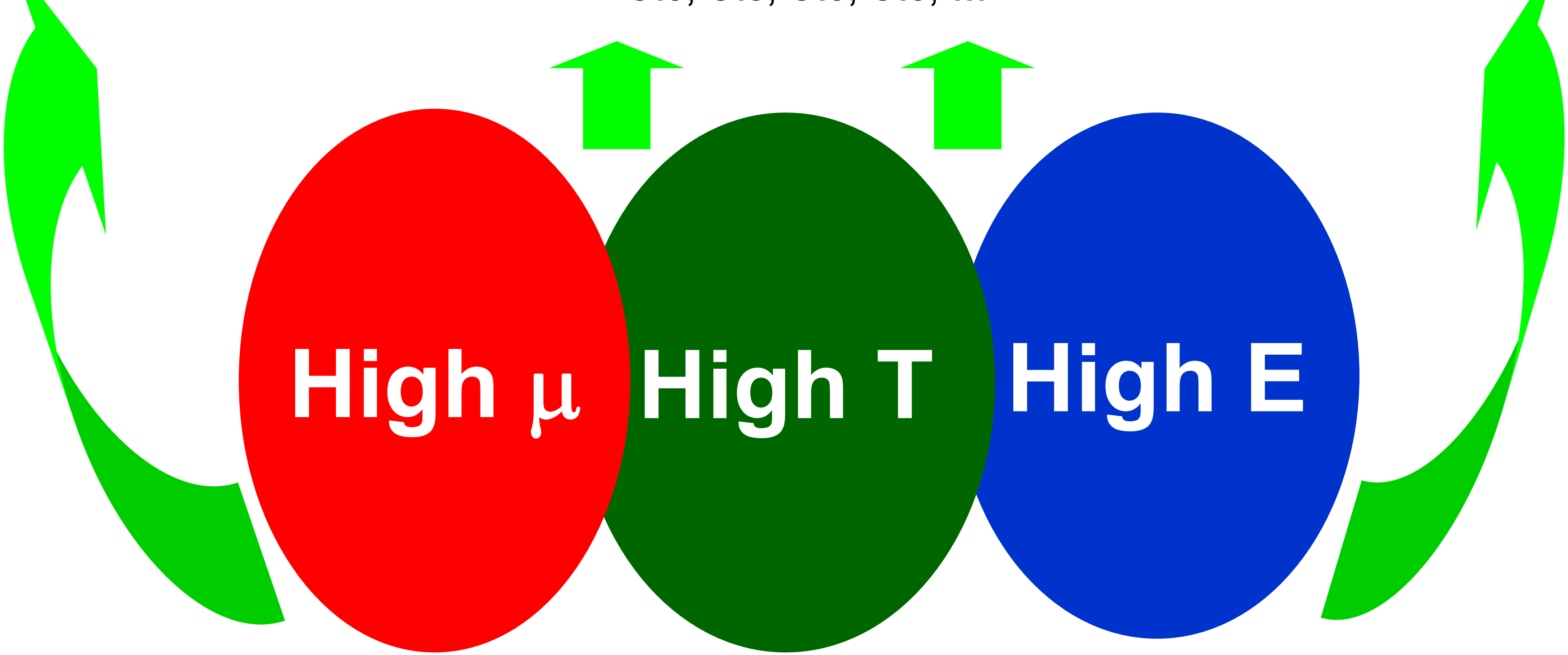
Extreme QCD – Faces

Color Superconductivity*, *Quark-Gluon Plasma*, *Color Glass Condensate

(Combined with Other Extremes) Strong Magnetic Field, Large N_c, Strong Coupling, etc

(With Various Methods) Lattice Simulation, Effective Model (**PNJL Model**), Gravity Dual

etc, etc, etc, etc, ...

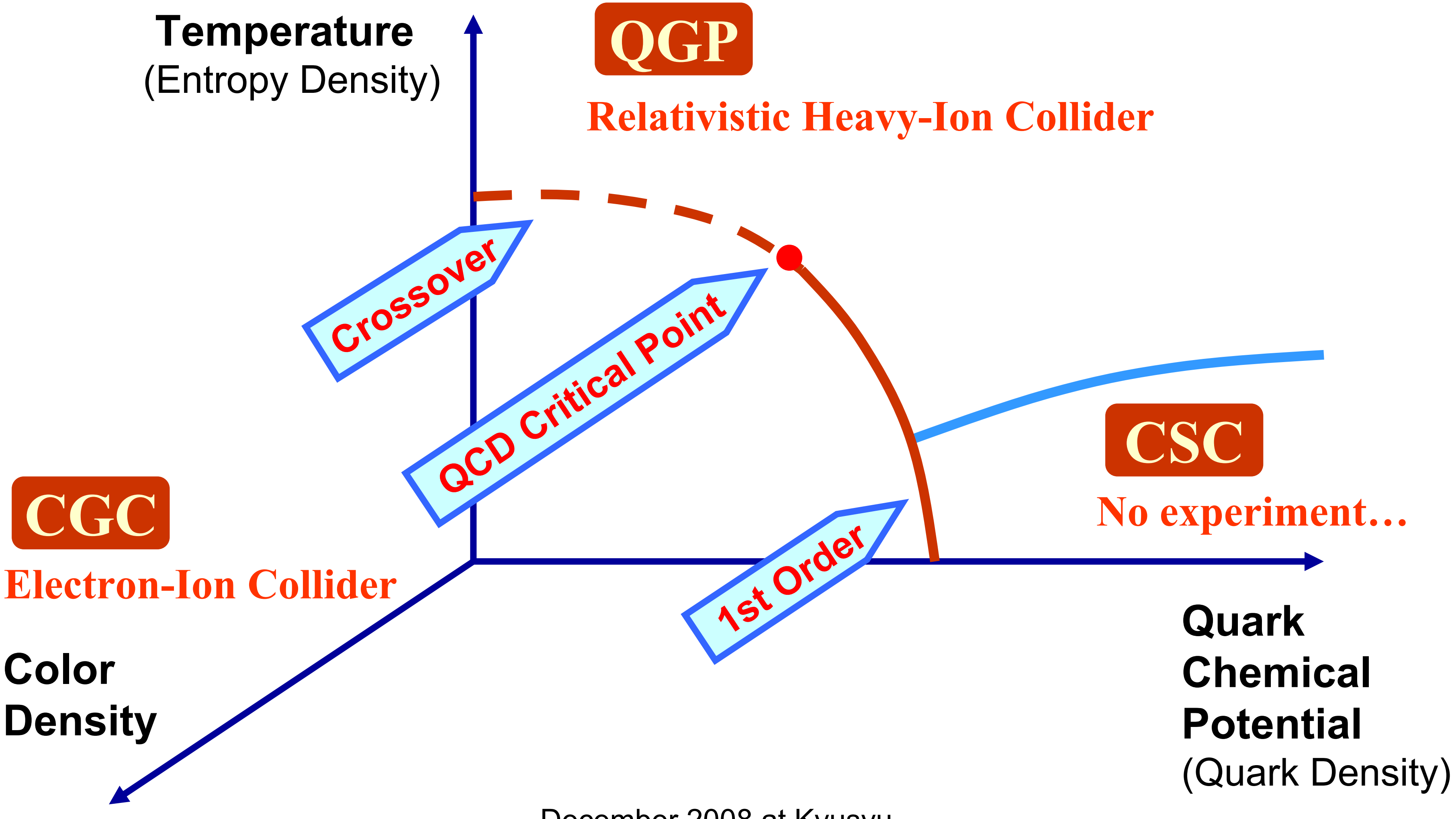


Two "Simplest"s

- ***Simplest* Questions**
  - What happens at extreme **high-density**?
  - What happens at extreme **high-temperature**?
  - What happens at extreme **high-energy**?
- ***Simplest* Answers**
  - Color Superconductivity (***CSC***)
  - Quark-Gluon Plasma (***QGP***)
  - Color Glass Condensate (***CGC***)

  } **Extreme QCD !**

QCD Paradigms



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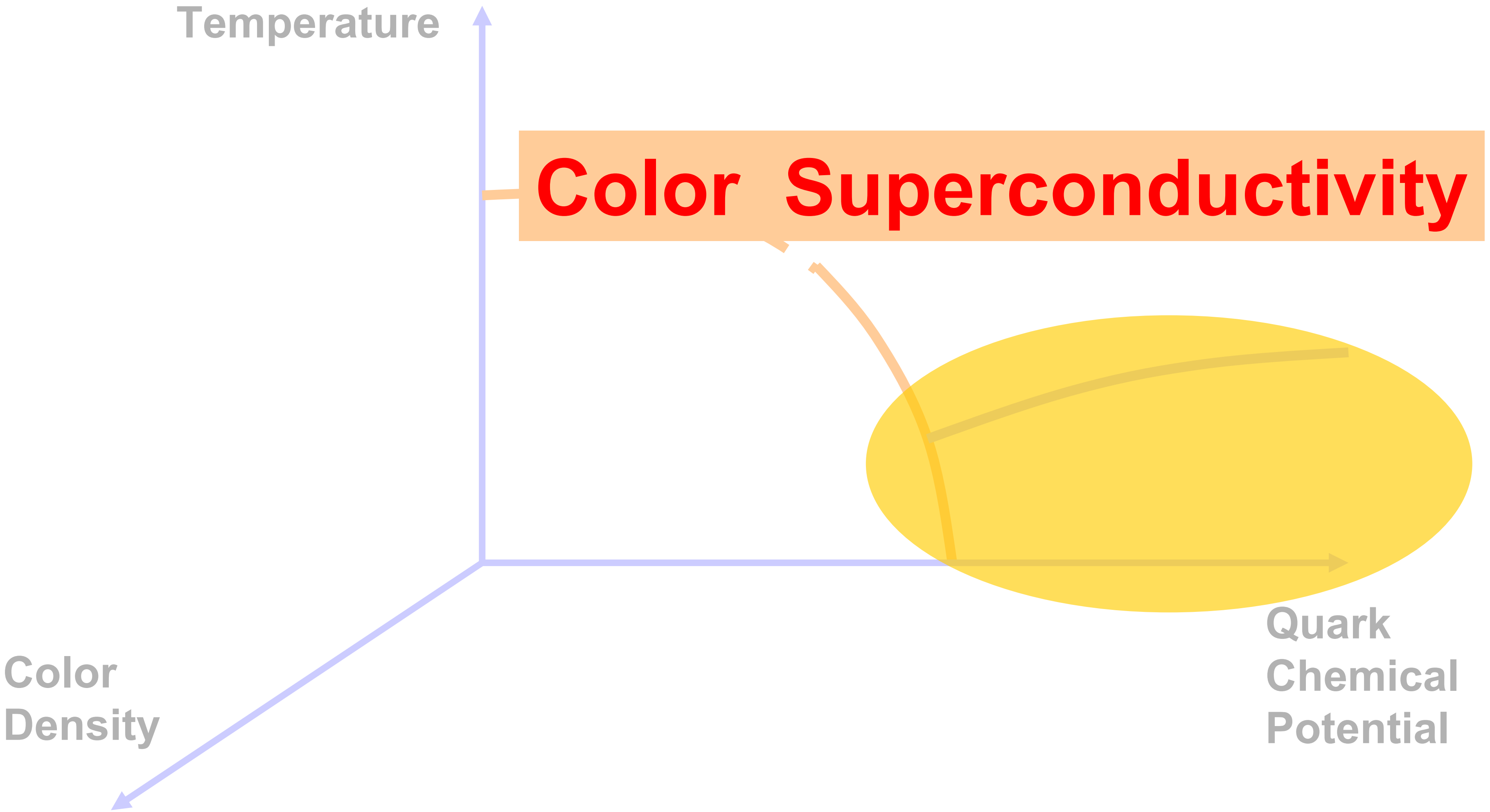
Dense Quark Matter



Key Issues

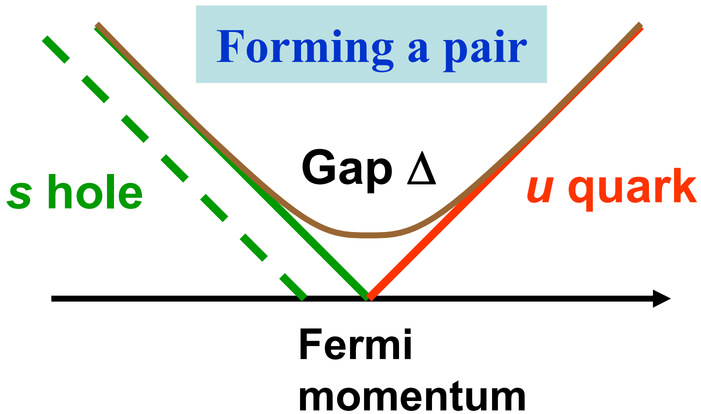
- **Color-Flavor Locked (CFL) Phase**
  - Ground state of three-flavor symmetric matter
- **Electric and Color Neutrality Conditions**
  - *s* breaks neutrality → Fermi surface mismatch
- **Gapless State ←→ Unstable**
- **Quest for the true ground state...???**
  - Crystalline CSC, LOFF, meson supercurrent CSC
  - Gluonic Phase

Superconductivity

Bailin-Love ('84)

Fermi surface $\mu_q \sim 500\text{MeV} \leftrightarrow \rho \sim 10\rho_0$

Attractive interaction $3+3 \to \bar{3}$



Color superconductivity is inevitable.

Many possible pairing patterns

Spin Color Flavor

Strange quark mass ~ 100 ~ 200MeV

Characteristic scale ~ m_s^2/μ_q (~50MeV)

$$\sqrt{p^2+m_s^2} - \mu_q \approx p - \mu_q + \frac{2m_s^2}{\mu_q} + \cdots$$

Breaking a pair

Color-Flavor Locked Phase

Order parameter

Alford-Rajagopal-Wilczek ('99)
Schafer-Wilczek ('99)
K.F. ('04)
Yamamoto *et al.* ('06)

$$2\left\langle \varepsilon_{ijk}\varepsilon_{\alpha\beta\gamma}\bar{q}_{j\beta}\gamma^5 q^{\mathrm{C}}_{k\gamma}\right\rangle = \left\langle \phi_{\mathrm{L}i\alpha}\right\rangle - \left\langle \phi_{\mathrm{R}i\alpha}\right\rangle$$

$$\left\langle \phi_{\mathrm{L}i\alpha}\right\rangle \quad [SU_C(3)] \times SU_L(3) \times U_L(1) \longrightarrow [SU_{C+L}(3)] \times Z_L(2)$$

$$\left\langle \phi_{\mathrm{R}i\alpha}\right\rangle \quad [SU_C(3)] \times SU_R(3) \times U_R(1) \longrightarrow [SU_{C+R}(3)] \times Z_R(2)$$

$$[SU_C(3)] \times SU_L(3) \times SU_R(3) \times U_V(1) \times U_A(1) \longrightarrow SU_{C+L+R}(3) \times Z_L(2) \times Z_R(2)$$

Gauge-invariant order parameter

$$\sigma \sim \left\langle \bar{\phi}_{\mathrm{L}}\phi_{\mathrm{R}}\right\rangle \qquad H \sim \varepsilon_{ijk}\varepsilon_{\alpha\beta\gamma}\left\langle \phi_{\mathrm{L}i\alpha}\phi_{\mathrm{L}j\beta}\phi_{\mathrm{L}k\gamma}\right\rangle$$

$$q \leftrightarrow \bar{\phi}, \quad \bar{q} \leftrightarrow \phi$$

Hadron $\leftrightarrow$ Quark (CFL)

Four-Fermi (NJL) Model

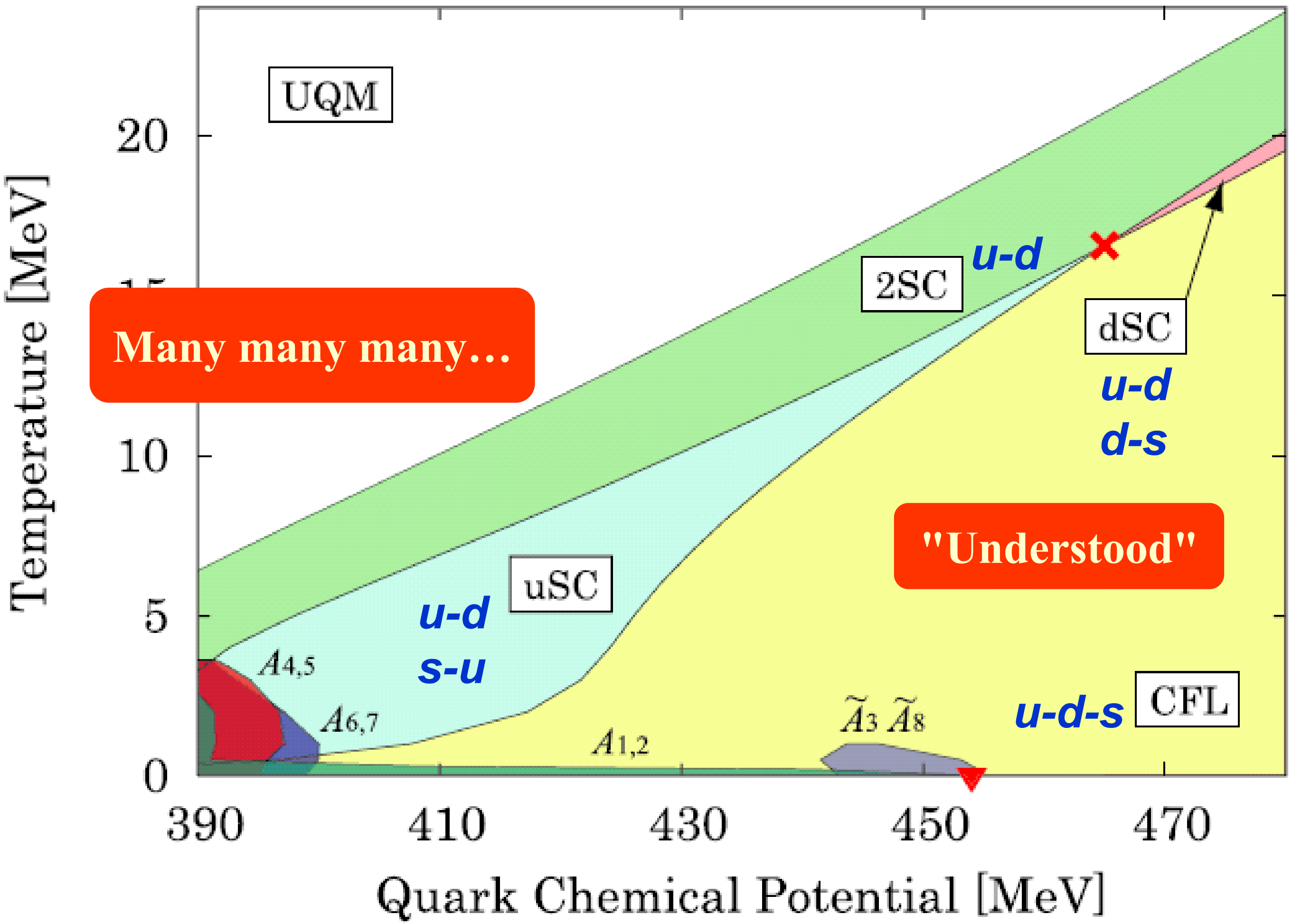
- **NOT useful** to estimate the pairing gap itself.
  - ♦ Gluon part of interaction is in $\Delta(g)$ **Pisarski-Rischke ('00)**

$$\Delta(g) = 2 \cdot 256\pi^4 \cdot (1/g^5)\, \mu_\mathrm{q}\, e^{-3\pi^2/\sqrt{2}g} \qquad (\mathrm{2SC}; \Delta \sim 0.1\mu_\mathrm{q} \text{ for } g^2/4\pi \sim 1)$$

- **Useful** to examine the property of dense quark matter with the pairing gap Δ **given**.
  - ♦ Mean-field quark propagator = QCD in the mean-field approx.
  - ♦ Only relevant interaction near the Fermi surface

***Assumptions*:**
Quasi-particle picture = Non-interacting quarks with Δ
Δ(2SC) ~ 50MeV around μ_q ~ 500MeV (c.f. $m_s^2/\mu_\mathrm{q} \sim 50\mathrm{MeV}$)

Robust Part (after chiral restoration)



From K.F. ('05)

Condensed Matter Physics of QCD is established.
Analogy to ^{3}He
e.g.)
Zero Sound, Phonon, etc
Hands et al. ('04)
Fukushima-Iida ('05)

Toward the True Ground State

(Chromomagnetic) Instability

Shovkovy-Huang ('04)
Casalbuoni et al. ('04)
Fukushima ('05)

$m_s^2/\mu_q > \Delta$ → Unstable w.r.t. $\Delta(q)$, A^T, $n(q)$

Pressure to tear the Cooper pair apart
Energy gain by condensation

Iida-Fukushima ('06)

Crystalline Color Superconductivity (LOFF)

$$(\partial - igA)\Delta = \partial\left(\Delta\, e^{-igA\cdot x}\right)$$

Giannakis-Ren ('04)

Larkin-Ovchinnikov ('65)
Fulde-Ferrell ('64)

One component gluon condensation == Plane-wave LOFF

Multi component = Gluonic Phase

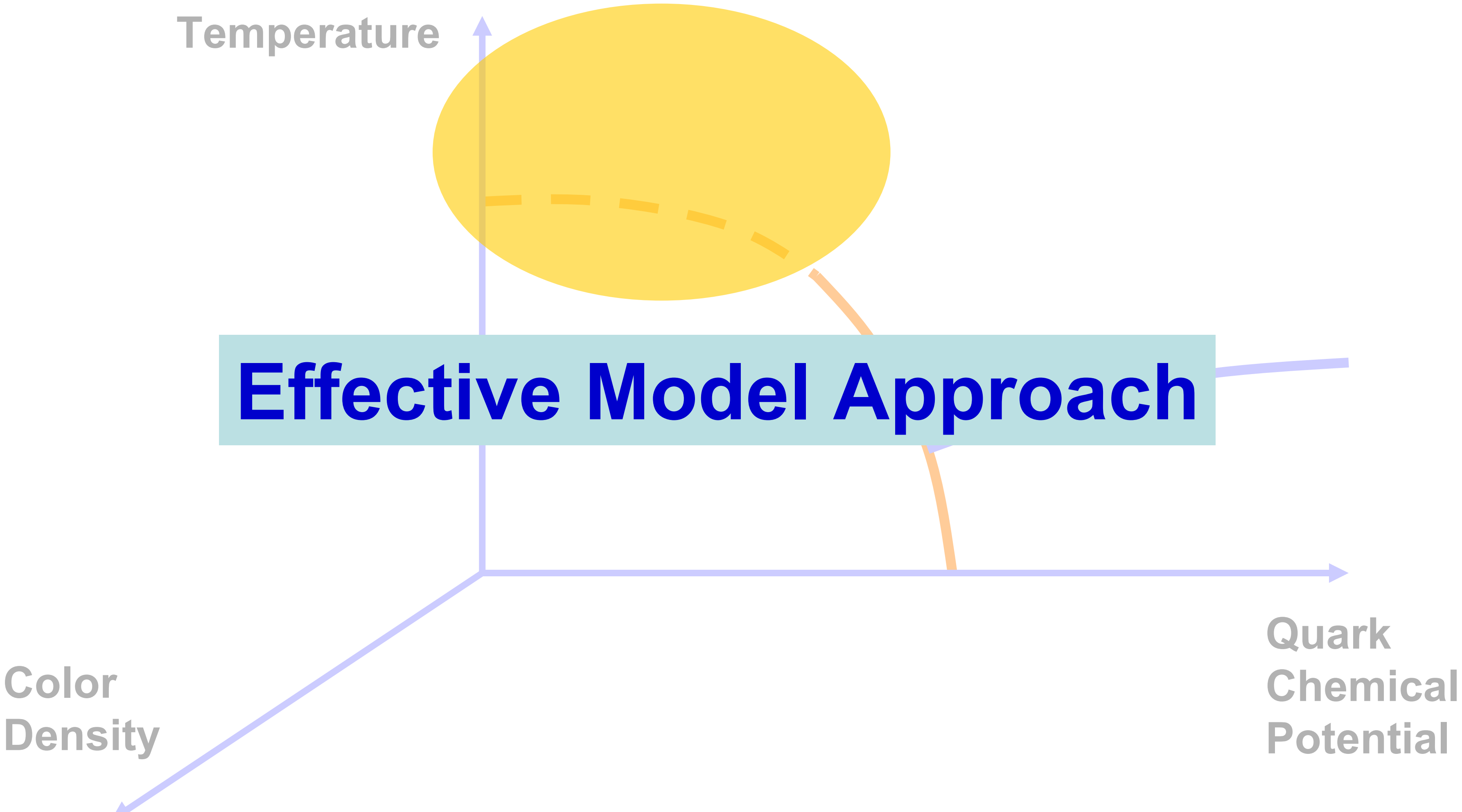
VS

Multiple-wave = Crystalline Phase

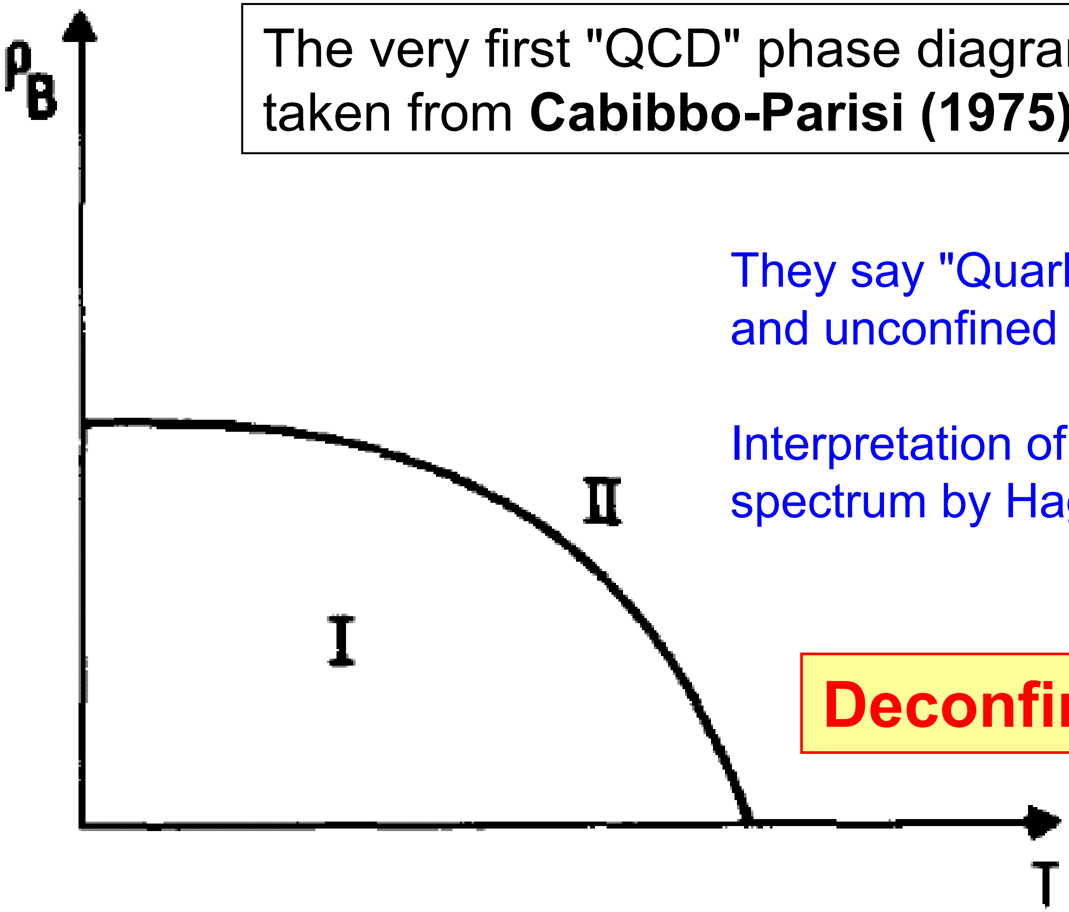
Gorbar-Hashimoto-Miransky ('06)

Alford-Bowers-Rajagopal ('01)
Rajagopal-Sharma ('06)
Alford *et al.* RMP80:1455 (2008)

Hot Quark (and Gluon) Matter



History of the Phase Diagram



The very first "QCD" phase diagram taken from **Cabibbo-Parisi (1975).**

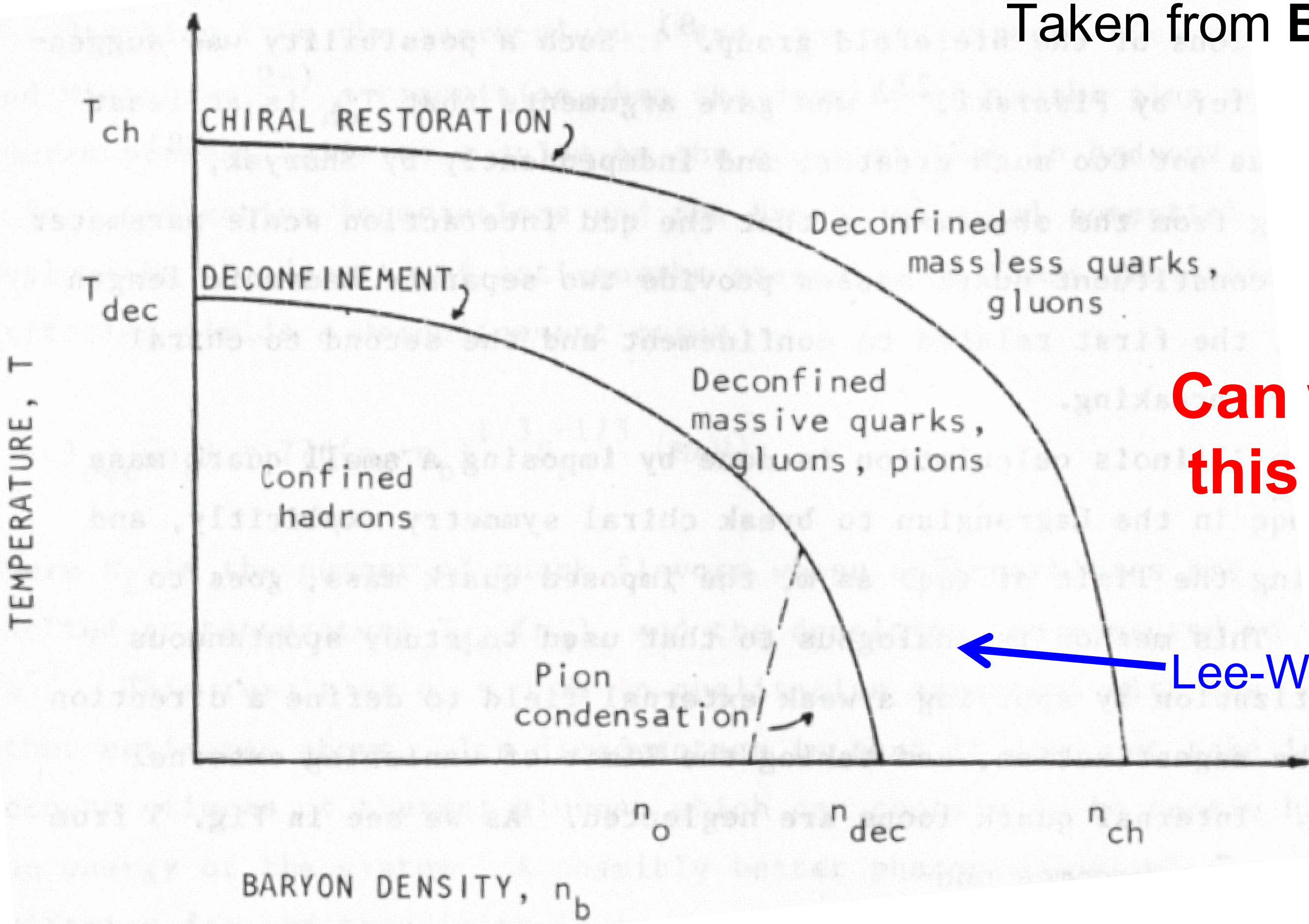
They say "Quarks are confined in phase I and unconfined in phase II".

Interpretation of the exponentially increasing spectrum by Hagedorn.

Deconfinement Transition

Bielefeld Proceedings in 1982

Taken from **Baym (1982)**



Can we exclude this possibility ???

Lee-Wick abnormal matter

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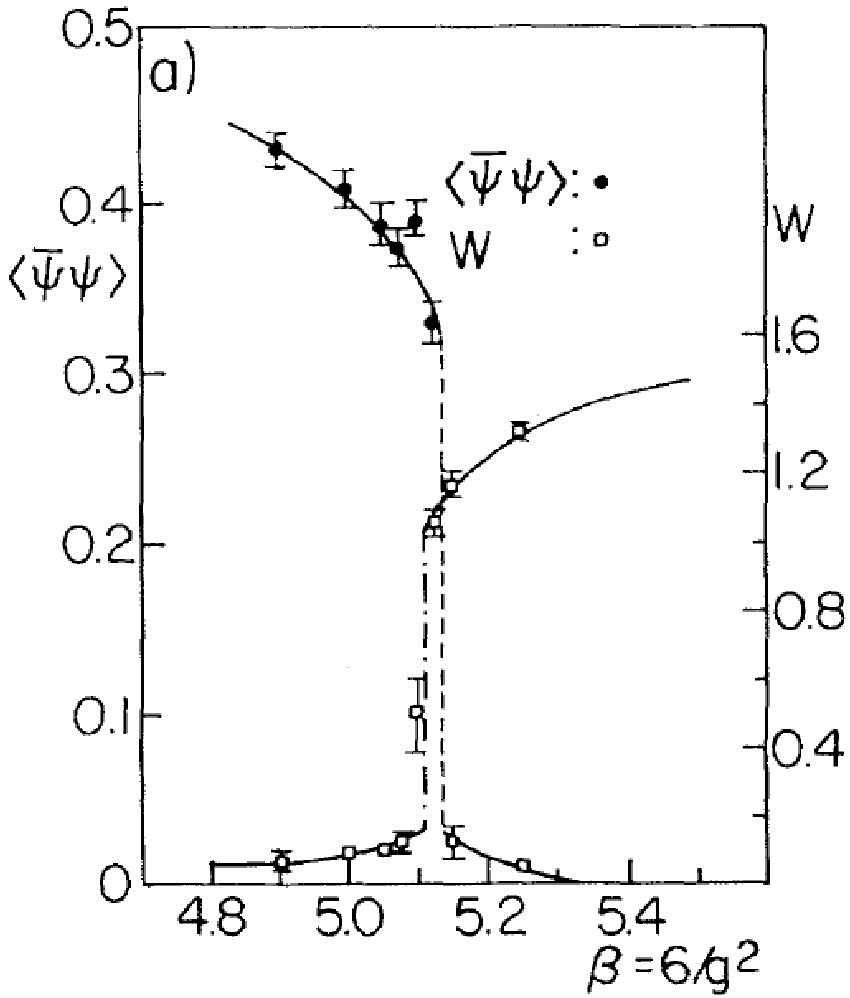
Lattice QCD



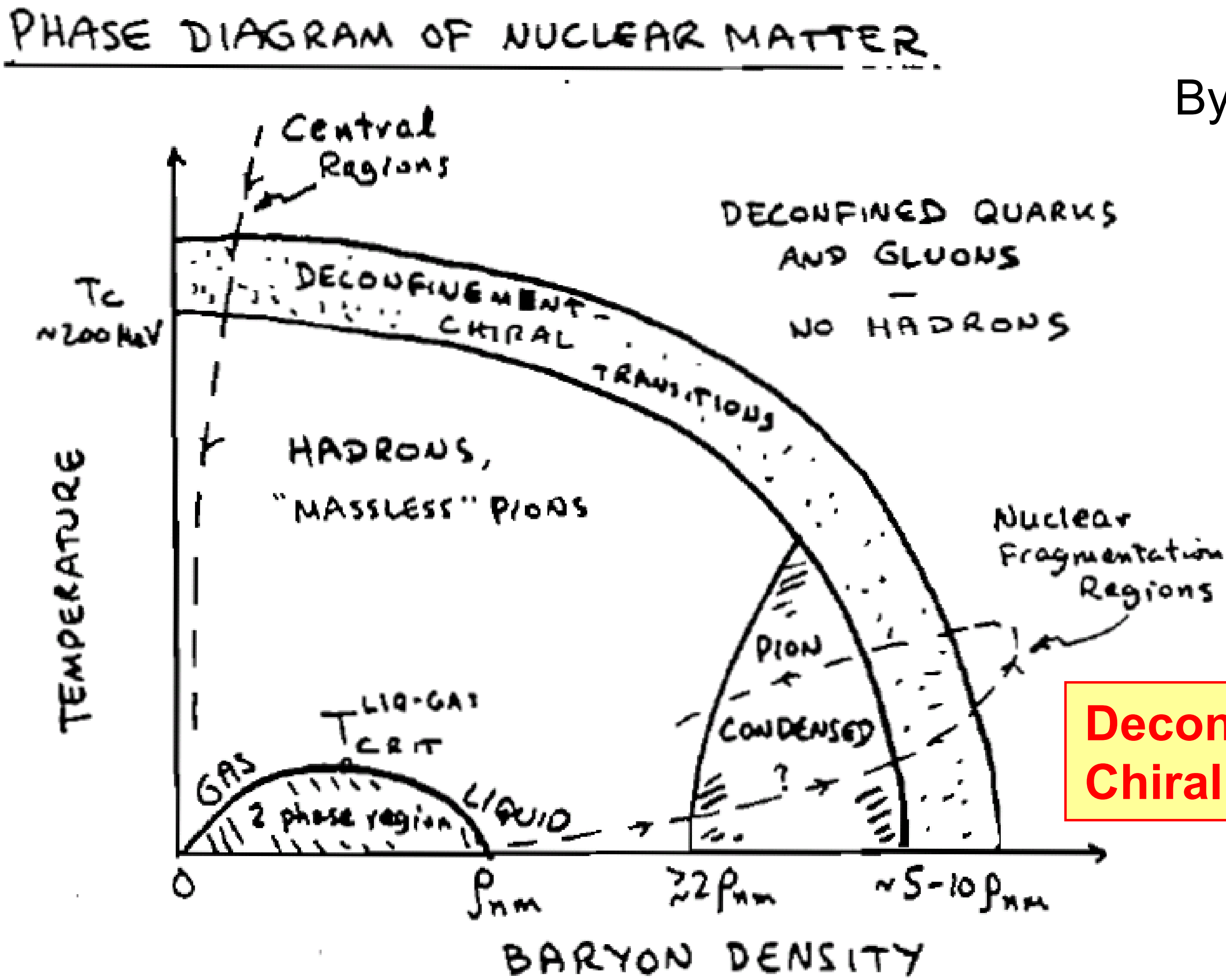
Taken from
Kogut, Stone, Wyld, Gibbs, Shigemitsu, Shenker, Sinclair (1983)

$\langle \overline{\psi}\psi \rangle$	Chiral Condensate
$W \sim \exp\left[-f_q / T\right]$	Polyakov Loop

Chiral Restoration occurs simultaneously with **Deconfinement !**

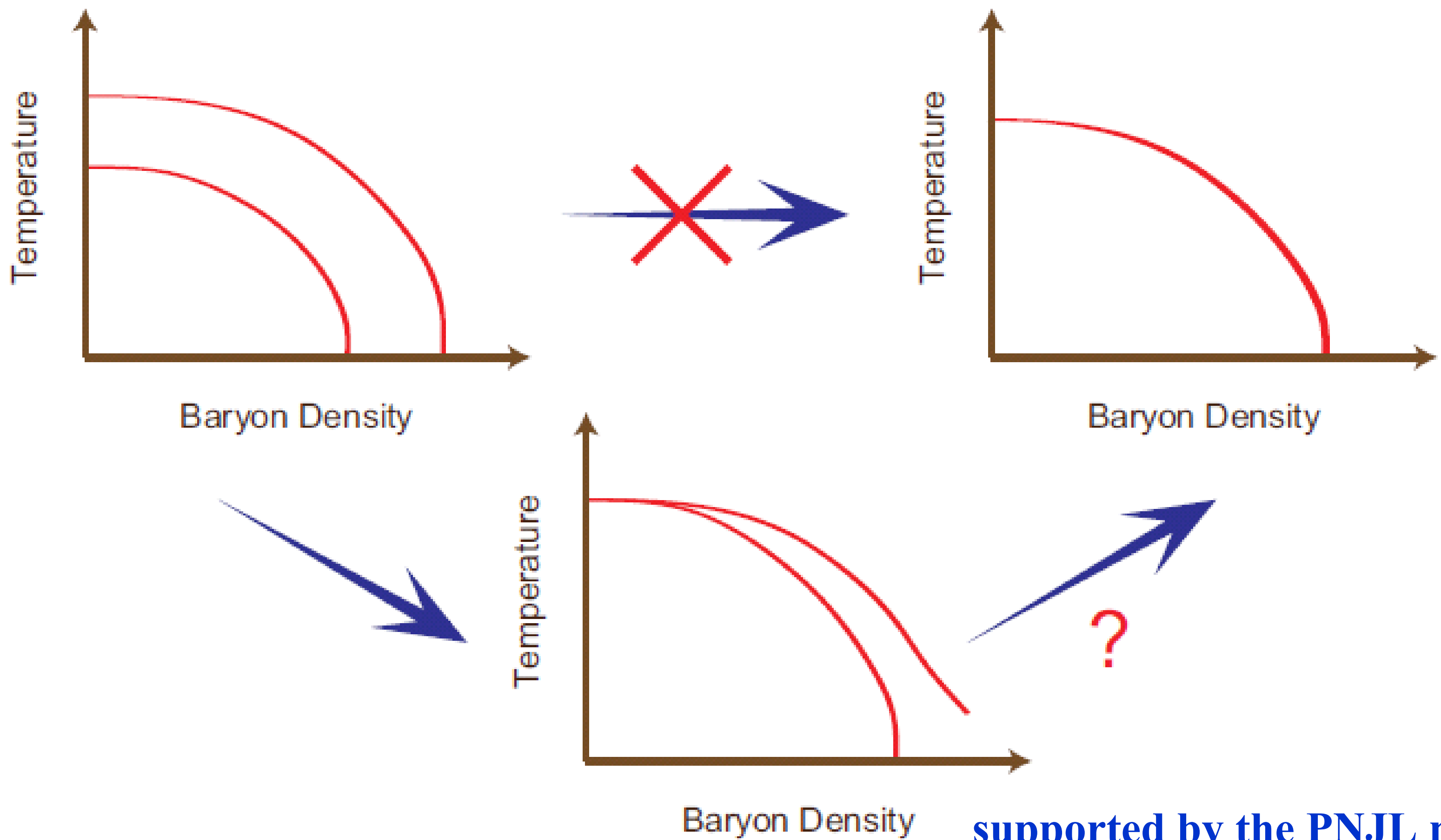
Long Range Plan in 1983

By **Baym (1983)**



Deconfinement and Chiral Transitions

Discontinuous Leap?



supported by the PNJL model

PNJL Model

$$\langle \mathrm{tr} L \rangle \sim \exp\left[-f_q / T\right] \sim 0$$ in the confinement phase

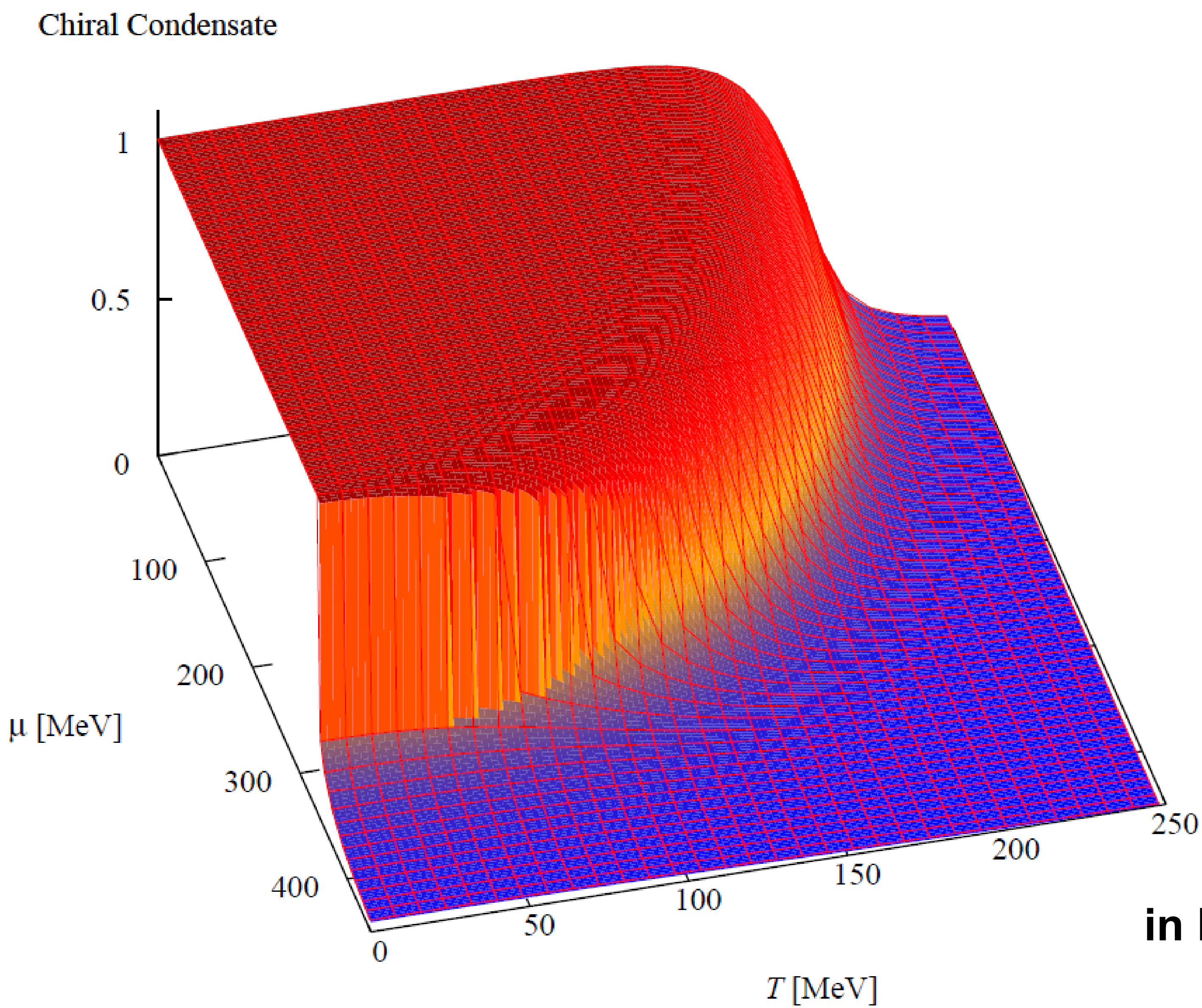
$$Z \sim \prod \left(1 + e^{-(\varepsilon-\mu)/T}\right)\left(1 + e^{-(\varepsilon+\mu)/T}\right)$$

$$\Rightarrow \prod \det\left(1 + L\, e^{-(\varepsilon-\mu)/T}\right)\left(1 + L^{+} e^{-(\varepsilon+\mu)/T}\right)$$

Only $L \cdot L^{+}$ and $L \cdot L \cdot L$ nonvanishing

meson-like **baryon-like**

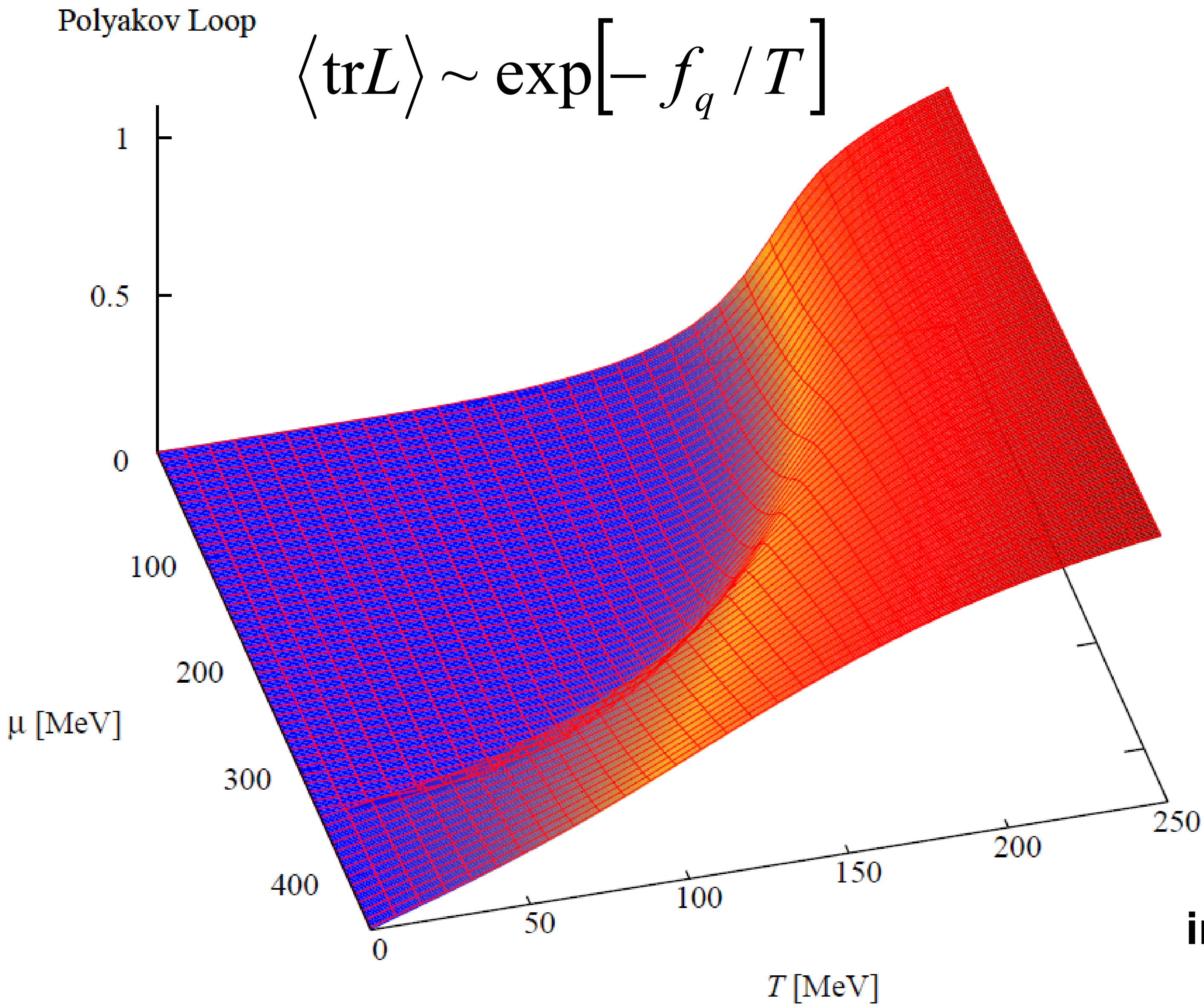
Chiral Condensate



in PNJL

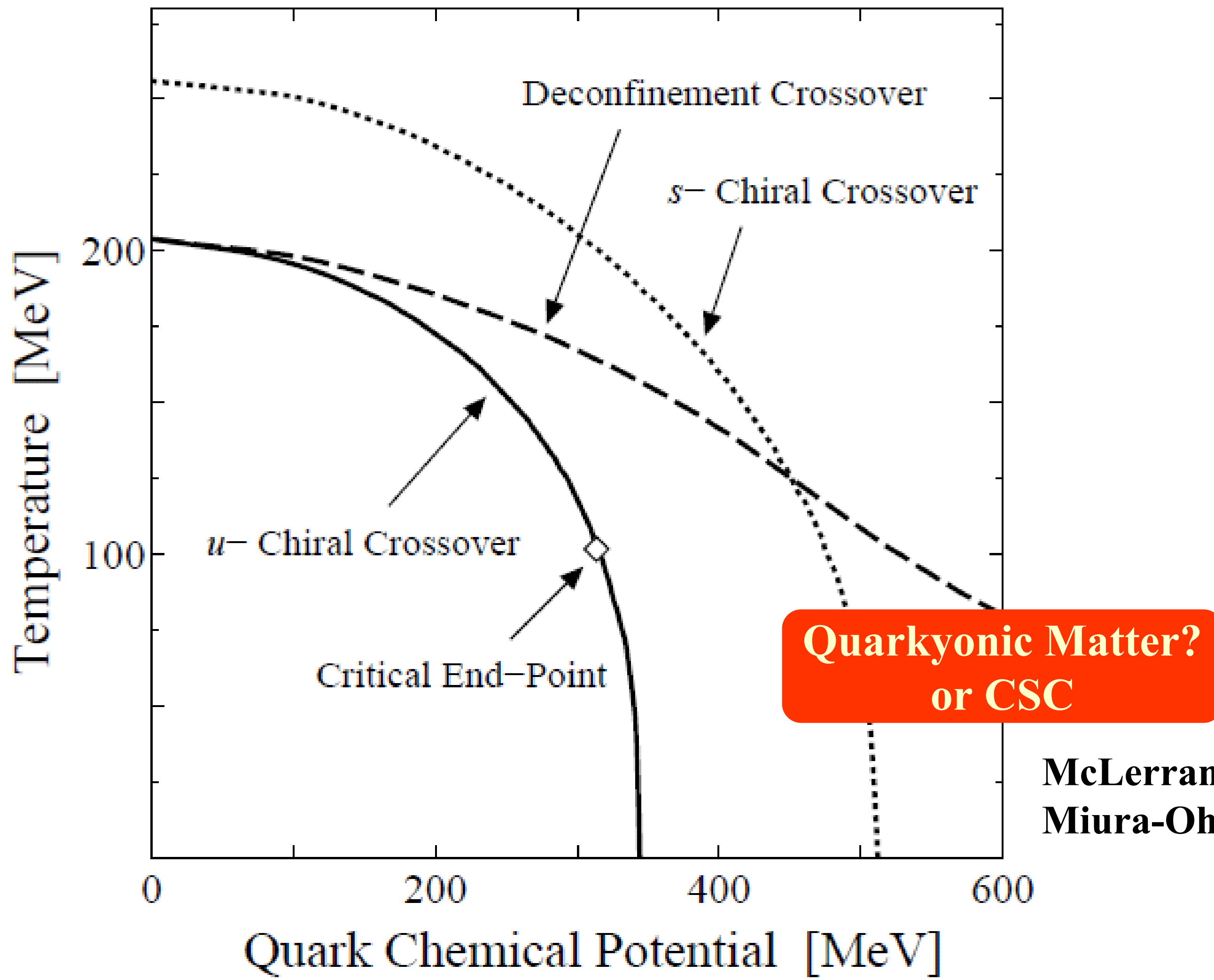
Polyakov Loop

$$\langle \mathrm{tr}L \rangle \sim \exp\left[-f_q / T\right]$$



in PNJL

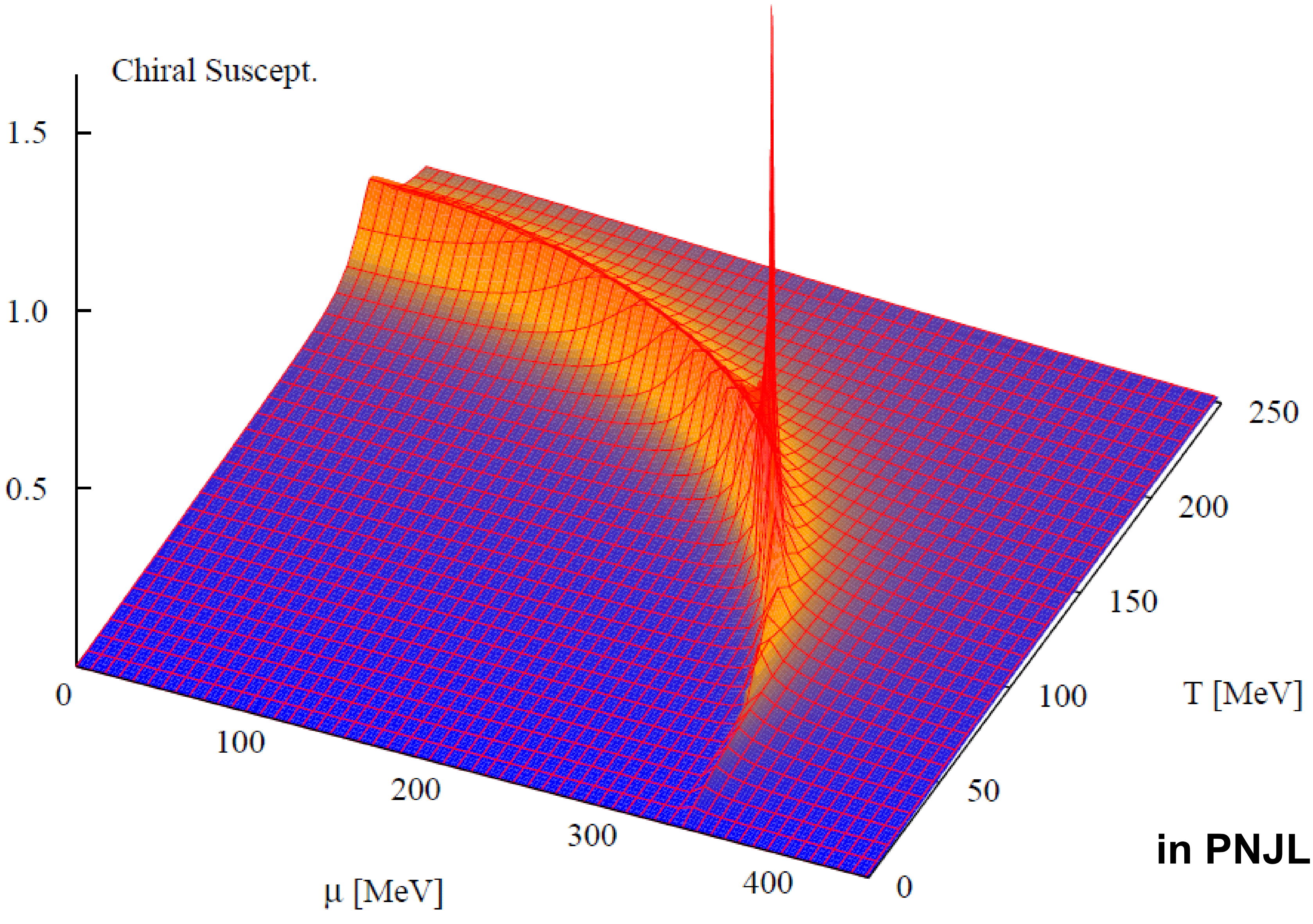
PNJL Phase Diagram



in PNJL

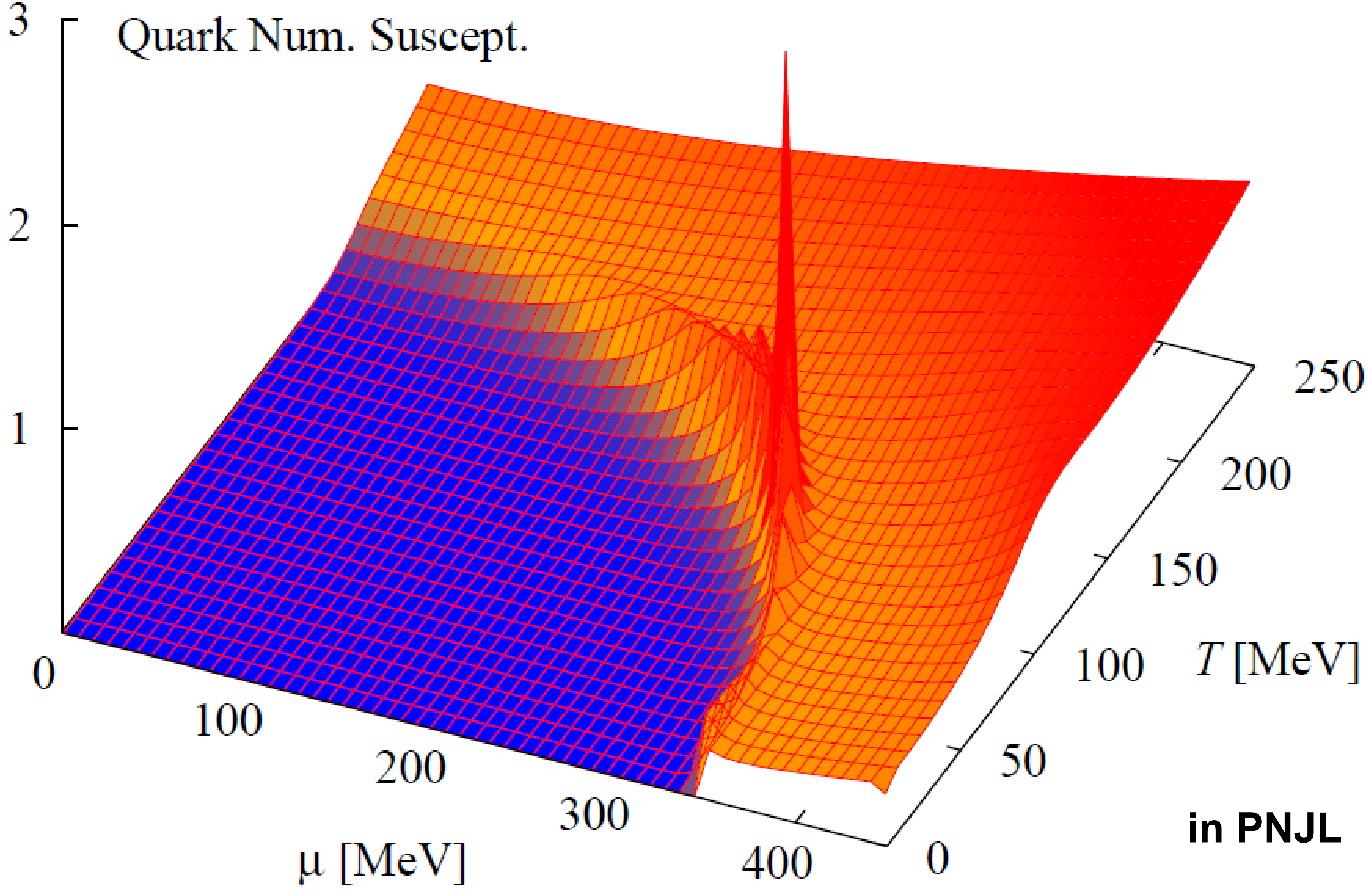
McLerran-Pisarski ('07)
Miura-Ohnishi ('08)

Chiral Susceptibility



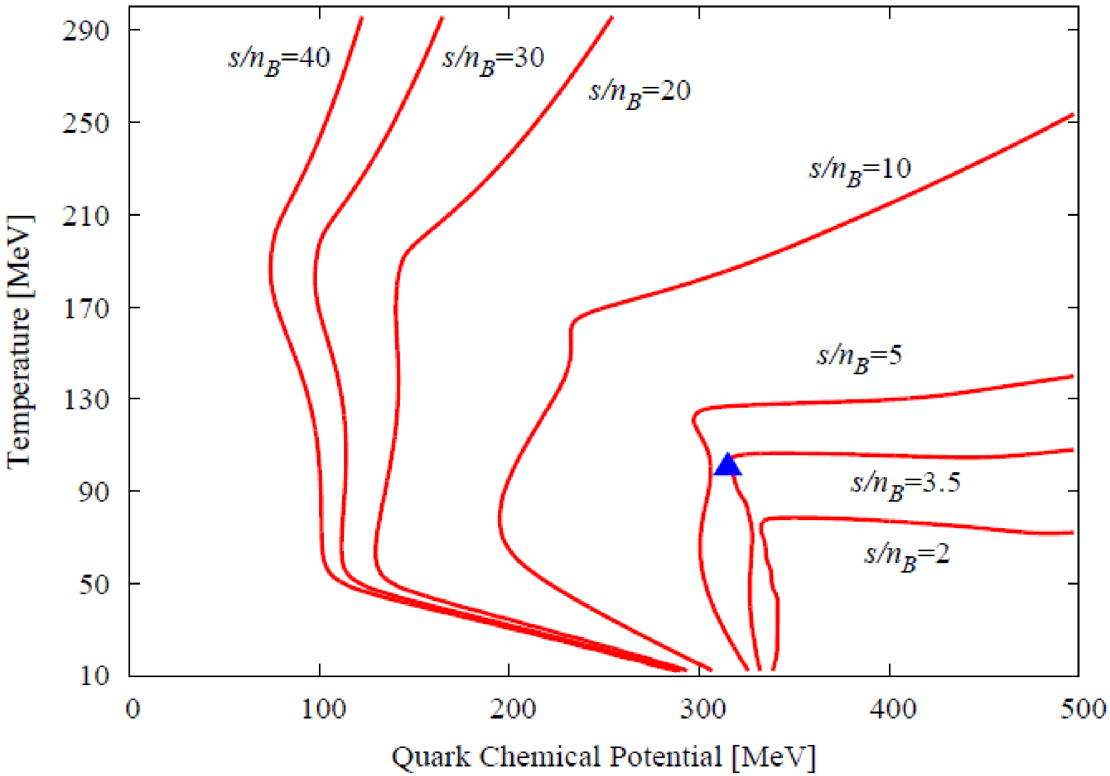
in PNJL

Quark Number Susceptibility

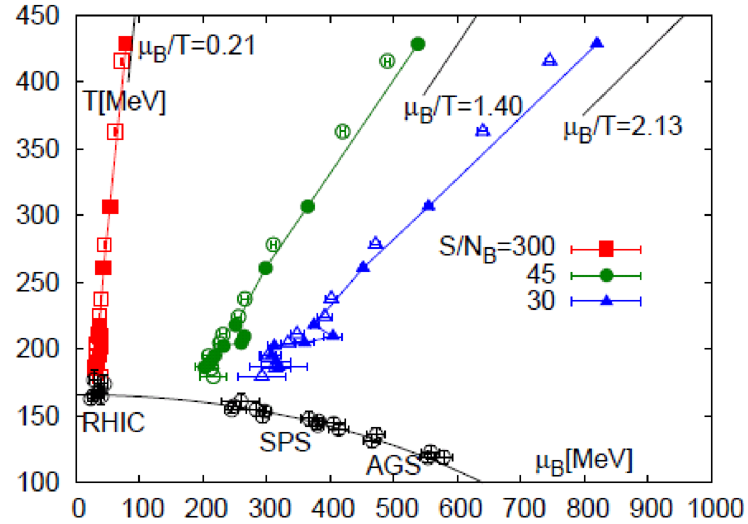


in PNJL

Adiabatic (Isentropic) Path



in PNJL



Schmidt ('08)

$$d(s/n_B)/d\tau = 0$$

Consistent with the lattice results so far.

As a "Toy" Model

Sign Problem

$$\ln Z \sim \ln \prod \det\left(1 + L e^{-(\varepsilon-\mu)/T}\right)\left(1 + L^{+} e^{-(\varepsilon+\mu)/T}\right)$$

$$\sim \int \underbrace{\left(\mathrm{tr} L e^{-(\varepsilon-\mu)/T} + \mathrm{tr} L^{+} e^{-(\varepsilon+\mu)/T}\right)}$$

Fukushima-Hidaka ('05)

Complex for general *L* !

Imaginary Chemical Potential

$$\int \left(\mathrm{tr} L e^{-(\varepsilon-\mu)/T} + \mathrm{tr} L^{+} e^{-(\varepsilon+\mu)/T}\right)$$

$$\to \int e^{-\varepsilon/T}\left(\mathrm{tr} L e^{i\mu/T} + \mathrm{tr} L^{+} e^{-i\mu/T}\right)$$

Real number !

Sakai-Kashiwa-Kouno-Yahiro ('08)

PNJL to CSC ?

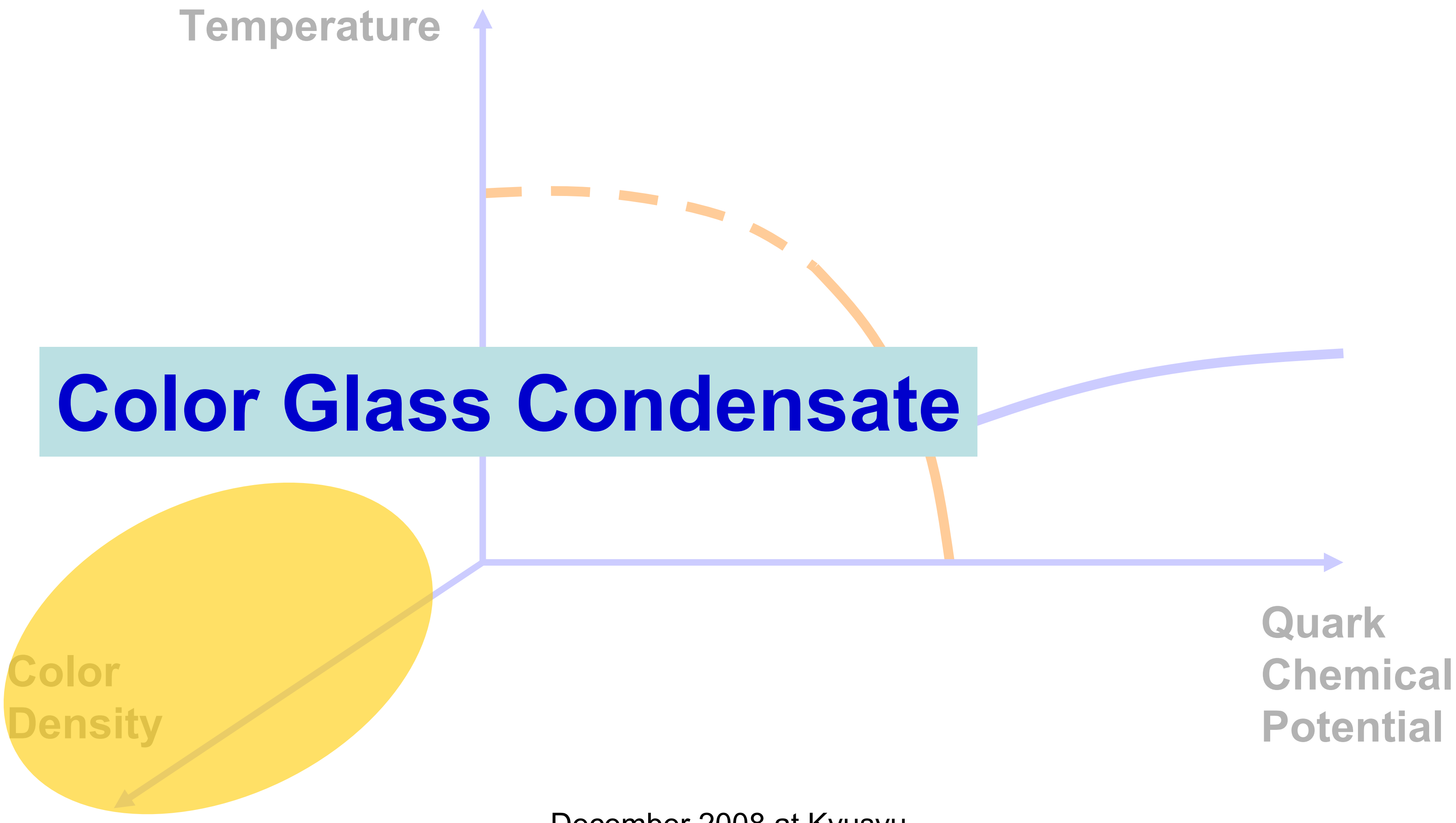
- There are already some applications of the PNJL model to Color Superconductivity.

Roessner-Ratti-Weise ('06)
Blaschke ('08)
Abuki et al ('08)

- There are some technical difficulties left unsolved, however.
  - How to compute the "color density"?
  - 2-flavor CSC incompatible with diagonal L .

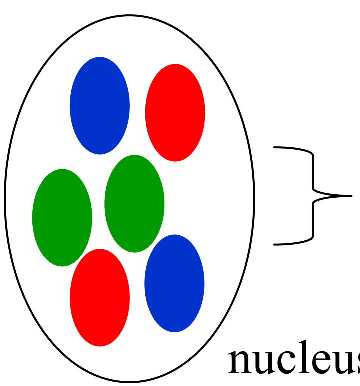
Abuki-Fukushima ('09) under completion

Dense Gluon Matter



How is dense color possible?

- **Large quark density → CSC** **Dense when $\mu_q > \Lambda_{QCD}$**
  - Quark or baryon # has a conserved U(1) charge.
- Chromo (Gluo)dynamics
  - Gauge charge is SU(3) without conservation.
  - Color is confined.
- **Large gluon density → Color Glass Condensate**



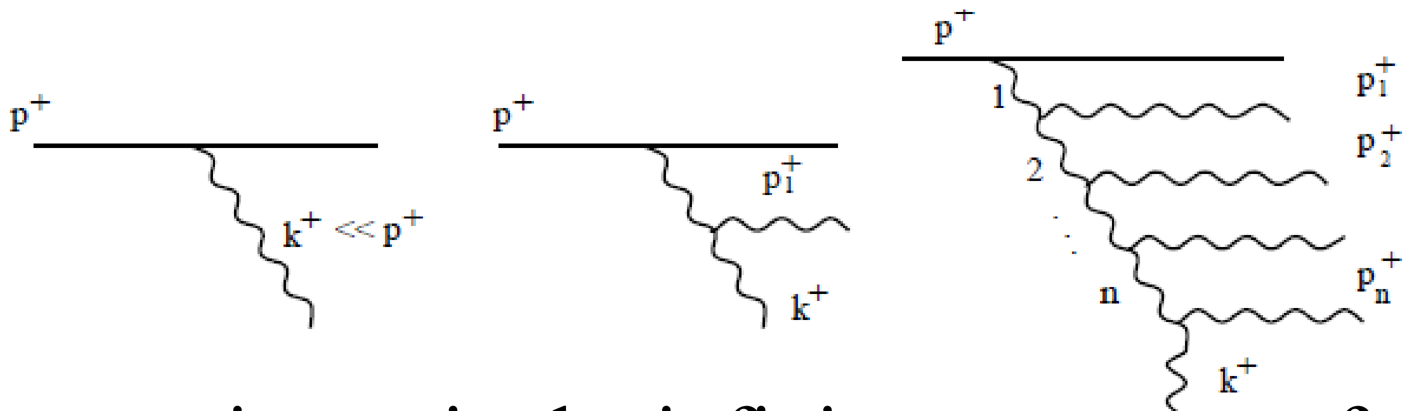
Parton overlapping
resolution ~ $1/Q_s$
$Q_s^2 \sim A^{1/3}$

nucleus ~A

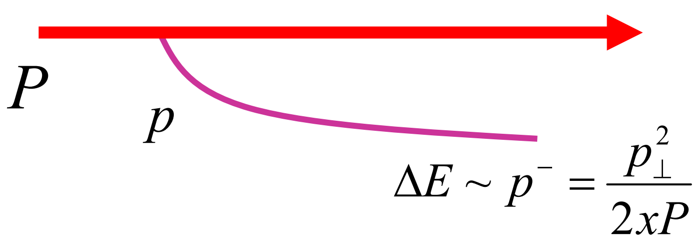
If seen with a probe at $1/Q >> 1/Q_s$
→ Dense!
If seen with a probe at $1/Q << 1/Q_s$
→ Dilute!

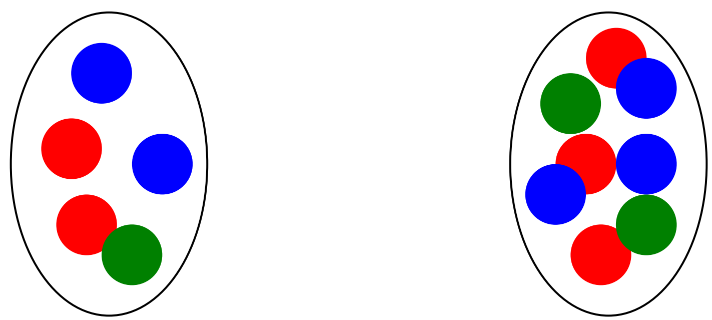
More Gluons from Quantum Radiation

- Gluon increases as going to higher energy



- Parton picture in the infinite momentum frame

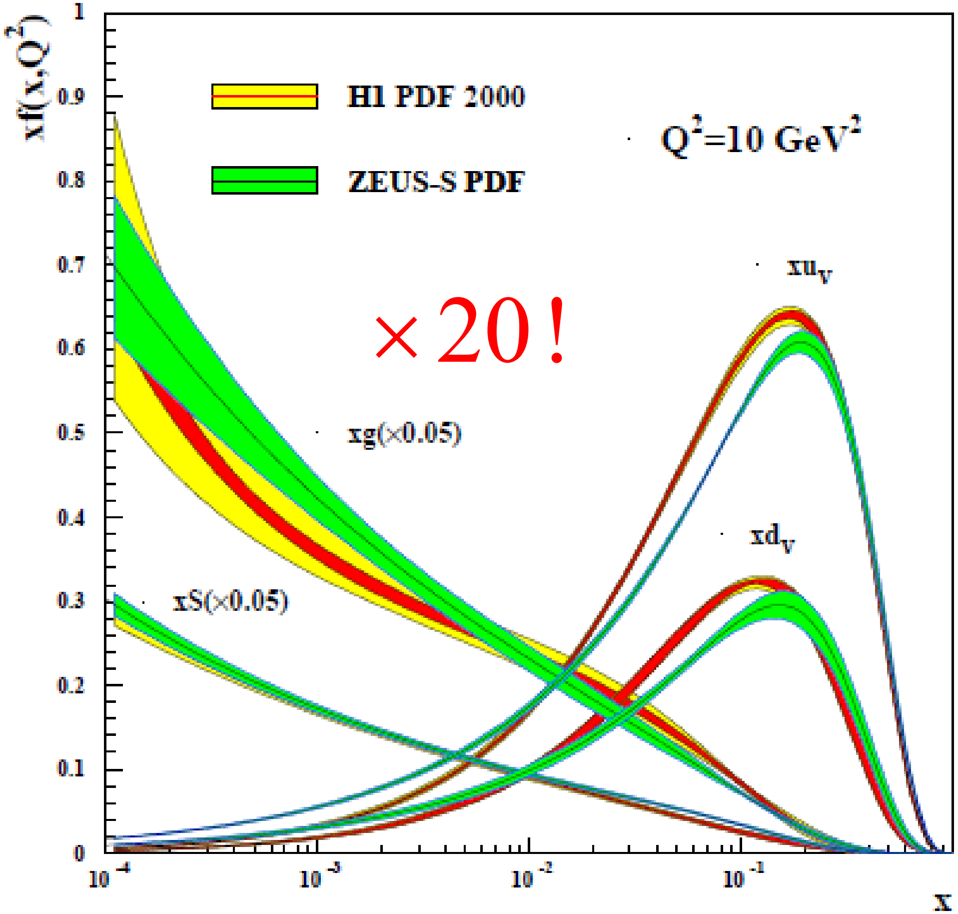




High Energy → Dense Gluon Matter

HERA (ep collider)

Quantum Evolution of PDFs



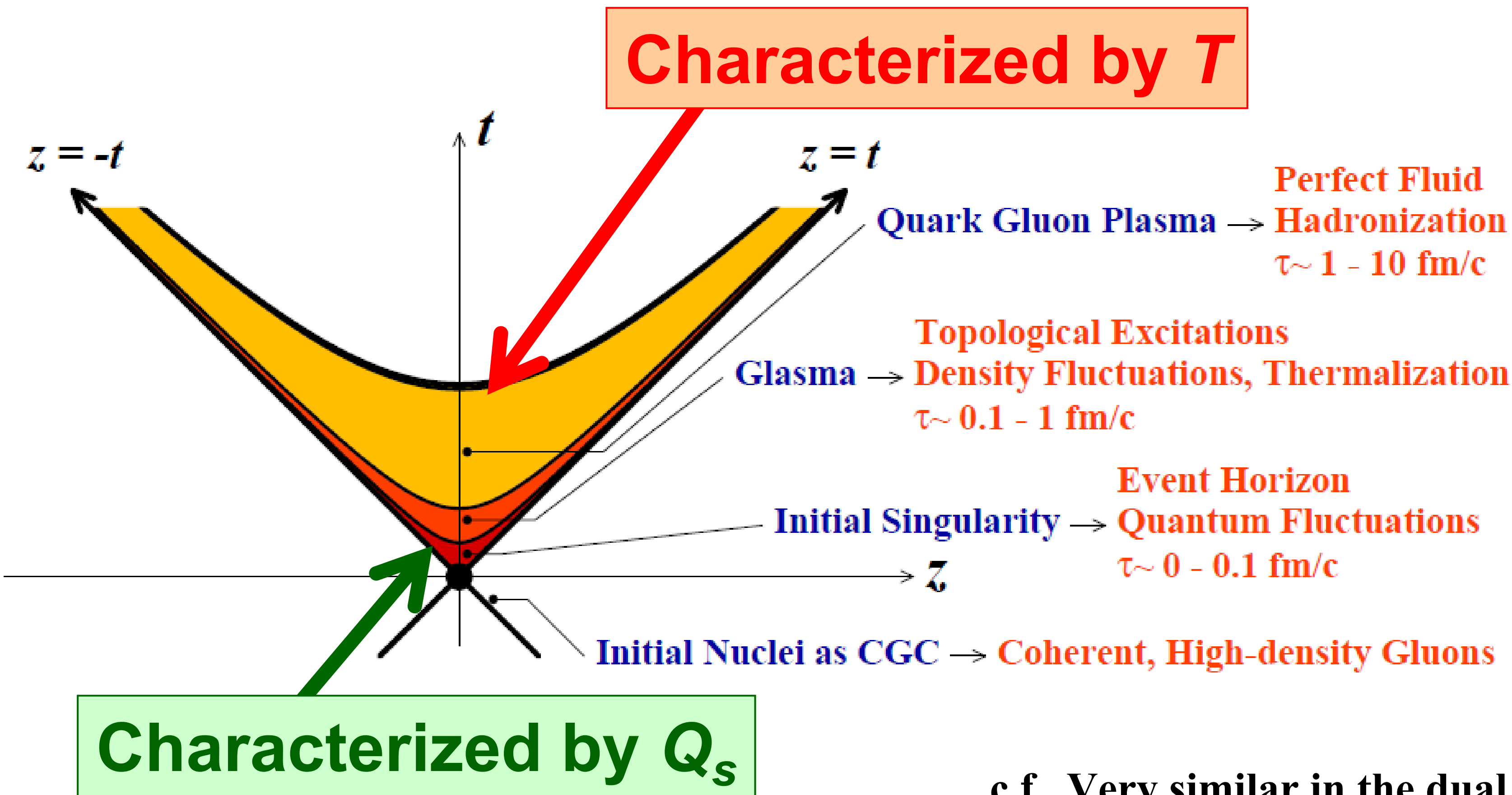
As x goes smaller than ~ 10^{-2}
gluon is dominant.

small-x = high energy

Only one energy scale

$\boldsymbol{Q_s}$

Initial Condition for HIC



Initial condition for HIC can be given!

c.f. Very similar in the dual gravity description.

Summary

- **Dense Quark Matter**
  - ***Color Superconductivity***
  - What is the true ground state?
- **Hot Quark (and Gluon) Matter**
  - ***Quark-Gluon Plasma***
  - PNJL model is successful!
- **Dense (and Hot) Gluon Matter**
  - ***Color Glass Condensate***
  - Initial condition for the heavy-ion collisions.

Various faces of ...

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